Discrete Mathematics (MATH-161)

`Relations, Recurrence Relations, Growth of Algorithms, & Counting Principles.`

(Assignment#2) **Dated:** 7^{th} **December 2015**

1 Relations:

1. Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.

2. R is the relation defined on $\mathbf{Z}$ as follows: For all $(m, n) \in \mathbf{Z}$, $mRn \Leftrightarrow 4|(m^2 + n^2)$.

3. Let $\mathbf{R}$ be the set of all real numbers and define a relation R on $\mathbf{R} \times \mathbf{R}$ as follows: For all (a, b) and (c, d) in $\mathbf{R} \times \mathbf{R}$,

$$(a, b)R(c, d) \Leftrightarrow \text{ either } a < c \text{ or both } a = c \text{ and } b \leq d.$$

   Is R a partial order relation? Prove or give a counterexample.

4. Define a relation R on the set $\mathbf{Z}$ of all integers as follows: For all $m, n \in \mathbf{Z}$, $mRn \Leftrightarrow m + n$ is even. Is R a partial order relation? Prove or give a counterexample.

5. Suppose R and S are antisymmetric relations on a set A. Must $R \cup S$ also be antisymmetric? Explain.

6. Suppose that R_1 and R_2 are equivalence relations on the set S. Determine whether each of these combinations of R_1 and R_2 must be an equivalence relation.

$$a)R_1 \cup R_2 \qquad b)R_1 \oplus R_2$$

7. Find the smallest relation containing the relation $\{(1, 2), (1, 4), (3, 3), (4, 1)\}$ that is
   (a) reflexive and transitive.
   (b) symmetric and transitive.
   (c) reflexive, symmetric, and transitive.

8. Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad = bc$. Show that R is an equivalence relation.

9. Let A be the set of all English statements. A relation I is defined on A as follows: For all $p, q \in A$, $pIq \Leftrightarrow p \rightarrow q$ is true. Show that A is an equivalence relation.

10. Define a relation R on the set $\mathbf{Z}$ of all integers as follows: For all $m, n \in \mathbf{Z}, mRn \Leftrightarrow m + n$ is even. Is R a partial order relation? Prove or give a counter-example.

11. ## 2 Recurrence Relations:

12. Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s. How many such bit strings are there of length five?

13. A computer system considers a string of decimal digits a valid codeword if it contains an even number of 0 digits. Thus 07903 and 68433 are valid codewords of length five while 86031 is not valid.

14. Consider the non-homogeneous equation $x_n = 10x_{n-1} - 25x_{n-2} + 8.5^n$, $x_0 = 6, x_1 = 10$.

15. A vending machine dispensing books of stamps accepts only one-dollar coins, $1 bills, and 5$ bills.

    (a) Find a recurrence relation for the number of ways to deposit n dollars in the vending machine, where the order in which the coins and bills are deposited matters.

    (b) What are the initial conditions?

    (c) How many ways are there to deposit \$10 for a book of stamps?

16. Find a recurrence relation to describe the amount left to pay on a loan of £10000, with interest charged at 1.5% per month and fixed monthly payment of £250.

17. Find a recurrence relation to describe the amount of water in a swimming pool of volume 7 50,000 liters if 0.05% per day is lost to evaporation but 350 liters is added daily.

3 Counting Principles:

18. How many bit strings with length not exceeding n, where n is a positive integer, consist entirely of 1s, not counting the empty string?

19. How many strings are there of four lowercase letters that have the letter x in them?

20. How many bit strings of length ten both begin and end with a 1?

21. How many positive integers between 2000 and 9999 inclusive

    (a) are divisible by 6 but not divisible by 2?

    (b) are odd?

    (c) have three distinct decimal digits and are odd?

    (d) are not divisible by 4 but divisible by 5?

22. How many positive integers between 2000 and 9999 inclusive

    (a) are divisible by 4 or 5?

    (b) are not divisible by either 5 or 4?

    (c) are divisible by 5 or 4 but not both?

    (d) are not divisible by 5?

    (e) are divisible by 3, 4 or 5?

    (f) are not divisible by 3, 4 and 5?

23. How many strings of four decimal digits

    (a) do not contain the same digit twice?

    (b) end with an even digit?

    (c) have exactly three digits that are 9s?

24. Find composite numbers between 2 to 50 (inclusive) using Principle of Inclusion and Exclusion. Also draw a Vein diagram.

25. Find composite numbers between 2 to 82 (inclusive) using Principle of Inclusion and Exclusion.

26. Pigeonhole Elementary School has 500 students. Show that at least two of them were born on the same day of the year.

27. There are 800,000 pine trees in a forest. Each pine tree has no more than 600,000 needles. Show that at least two trees have the same number of needles.

28. There are five points inside an equilateral triangle of side length 2. Show that at least two of the points are within 1 unit distance from each other.

29. How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16?

Submission Date: 22^{nd} December 2015 (Before **2 p.m** !)

Instructions

1. Solve each question on the sperate sheet.

2. Write formula used in each question.

3. Draw tables where they are necessary.

4. Submit one soft and one hard copy of the solution.

5. No rewards will be given for an assignment submitted after due date.
6. **Second Assignment Quiz will be held in last week of Dec 2015.**
7. Each student will solve two questions. One on which he/she appears in attendance list (provided to CR of your class). Second question number is equal your first question + 10. Count of question number starts from 1 after 29.