Discrete Mathematics (MATH-161)

`Logics, Logical Operators, Conditional Statements, Logical Equivalences, Predicates & Quantifier, Nested Quantifiers, Rules of Inferences, Mathematical Induction.`

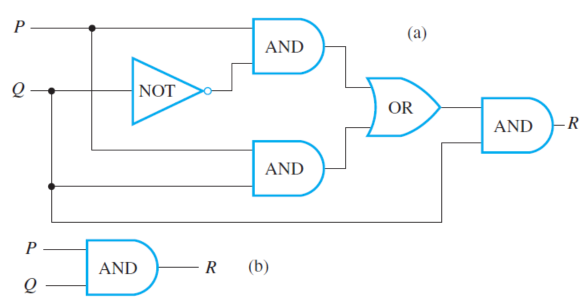
(Assignment#1) **Dated: October 25, 2015**

1. Write each of these statements in the form if p, then q in English.
   (a) To be a citizen of this country, it is sufficient that you were born in the United States.
   (b) That you get the job implies that you had the best credentials.
   (c) It is necessary to have a valid password to log on to the server.
   (d) You will reach the summit unless you begin your climb too late.

2. The logician Raymond Smullyan describes an island containing two types of people: knights who always tell the truth and knaves who always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusions?
   (a) A says: B is a knight.
   B says: A and I are of opposite type.
   (b) A says "The two of us are both knights" and B says "A is a knave."

3. When three professors are seated in a restaurant, the hostess asks them: Does everyone want coffee? The first professor says: I do not know. The second professor then says: I do not know. Finally, the third professor says: No, not everyone wants coffee. The hostess comes back and gives coffee to the professors who want it. How did she figure out who wanted coffee?

Logical Equivalence



4. Find the Boolean expressions for each circuit in the above figure. Show that these expressions are logically equivalent when regarded as statement forms.

5. By indicating the rule used at each step:

a- Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

b- Show that $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are logically equivalent.

c- Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

d- Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent.

e- Show that $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ are logically equivalent.

f- Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

Rules of Inference

6. Some of the arguments are valid, whereas others exhibit the converse or the inverse error. Use symbols to write the logical form of each argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise, state whether the converse or the inverse error is made.

   (a) If there are as many rational numbers as there are irrational numbers, then the set of all irrational numbers is infinite.
   The set of all irrational numbers is infinite.
   $\therefore$ There are as many rational numbers as there are irrational numbers.

7. The famous detective Percule Hoirot was called in to solve a baffling murder mystery. He determined the following facts:

   (a) Lord Hazelton, the murdered man, was killed by a blow on the head with a brass candlestick.

   (b) Either Lady Hazelton or a maid, Sara, was in the dining room at the time of the murder

   (c) If the cook was in the kitchen at the time of the murder, then the butler killed Lord Hazelton with a fatal dose of strychnine.

   (d) If Lady Hazelton was in the dining room at the time of the murder, then the chauffeur killed Lord Hazelton.

   (e) If the cook was not in the kitchen at the time of the murder, then Sara was not in the dining room when the murder was committed.

   (f) If Sara was in the dining room at the time the murder was committed, then the wine steward killed Lord Hazelton.

   Is it possible for the detective to deduce the identity of the murderer from these facts? If so, who did murder Lord Hazelton? (Assume there was only one cause of death.)

8. In set of premises and a conclusion are given. Use the valid argument forms to deduce the conclusion from the premises, giving a reason for each step. Assume all variables are statement variables

   (a) i. $\sim p \vee q \rightarrow r$

ii. $\sim q \vee s$

iii. $\sim t$

iv. $p \rightarrow t$

v. $\sim p \wedge r \rightarrow \sim s$

vi. $\therefore \quad \sim q$

(b) i. $p \vee q$

ii. $q \rightarrow r$

iii. $p \wedge s \rightarrow t$

iv. $\sim r$

v. $\sim q \rightarrow u \wedge s$

vi. $\therefore \quad \sim t$

Quantifiers and Predicates

9. Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase It is not the case that.)

(a) No one has lost more than one thousand dollars playing the lottery.

(b) There is a student in this class who has chatted with exactly one other student.

(c) No student in this class has sent e-mail to exactly two other students in this class.

(d) Some student has solved every exercise in this book.

(e) No student has solved at least one exercise in every section of this book

10. Let D be the set of all students at your school, and let M(s) be "s is a math major," let C(s) be "s is a computer science student," and let E(s) be "s is an engineering student." Express each of the following statements using quantifiers, variables, and the predicates M(s),C(s), and E(s).

(a) There is an engineering student who is a math major.

(b) Every computer science student is an engineering student.

(c) No computer science students are engineering students.

(d) Some computer science students are also math majors.

(e) Some computer science students are engineering students and some are not.

Rules of Inference for Quantifiers

11. Reorder the premises in each of the arguments to show that the conclusion follows as a valid consequence from the premises. It may be helpful to rewrite the statements in if-then form and replace some statements by their contrapositives.

- 1. When I work a logic example without grumbling, you may be sure it is one I understand.

- 2. The arguments in these examples are not arranged in regular order like the ones I am used to.
- 3. No easy examples make my head ache.
- 4. I canft understand examples if the arguments are not arranged in regular order like the ones I am used to.
- 5. I never grumble at an example unless it gives me a headache.

  ∴ These examples are not easy.

12. Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \wedge Q(x))$ is true then $\forall xP(x) \wedge \forall xQ(x)$ is true.

    (a) $\forall x(P(x) \wedge Q(x))$ Premise

    (b) $P(c) \wedge Q(c)$ Universal instantiation from (a)

    (c) $P(c)$ Simplification from (b)

    (d) $\forall xP(x)$ Universal generalization from (c)

    (e) $Q(c)$ Simplification from (b)

    (f) $\forall xQ(x)$ Universal generalization from (e)

    (g) $\forall xP(x) \wedge \forall xQ(x)$ Conjunction from (d) and (f)

13. Use resolution to show that the hypotheses "It is not raining or Yvette has her umbrella," "Yvette does not have her umbrella or she does not get wet," and "It is raining or Yvette does not get wet" imply that "Yvette does not get wet."

14. Indicate whether the following arguments are valid or invalid. Support your answers by converting these to symbolic form using laws of inference.

    (a) All teachers occasionally make mistakes.
    No gods ever make mistakes.
    ∴ No teachers are gods.

    (b) 1. When I work a logic example without grumbling, you may be sure it is one I understand.
    2. The arguments in these examples are not arranged in regular order like the ones I am used to.
    3. No easy examples make my head ache.
    4. I can't understand examples if the arguments are not arranged in regular order like the ones I am used to.
    5. I never grumble at an example unless it gives me a headache.
    ∴ These examples are not easy.

Methods of Proof

15. The square of any odd integer has the form 8m + 1 for some integer m.

16. Prove that $\sqrt{5}$ is not a rational number (prove by contradiction).

17. The quotient of any two rational numbers is a rational number.

18. Write the following numbers in ratio form to show that they are rational;
 0.56565656... 52.4672167216721...

19. Find mistake in proof of following theorems:

 (a) **Theorem:** The difference between any odd integer and any even integer is odd.
 "**Proof:** Suppose n is any odd integer, and m is any even integer. By definition of odd, n = 2k +1 where k is an integer, and by definition of even, m = 2k where k is an integer. Then n - m = (2k + 1) - 2k = 1. But 1 is odd. Therefore, the difference between any odd integer and any even integer is odd.Theorem: The difference between any odd integer and any even integer is odd. Proof: Suppose n is any odd integer, and m is any even integer. By definition of odd, n = 2k +1 where k is an integer, and by definition of even, m = 2k where k is an integer. Then n - m = (2k + 1) - 2k = 1. But 1 is odd. Therefore, the difference between any odd integer and any even integer is odd."

20. Prove that the statement, There exists an integer $k \geq 4$ such that $2k^2 - 5k + 2$, is prime.

Mathematical Induction

Use Mathematical induction to prove the following statements.

21. For every natural number n, $n(n^2 + 5)$ is a multiple of 6

22. $5^{2n-1} + 1$ is divisible by 6 for $n \in \mathbf{Z}^+$.

23. $\sum_{i=1}^{n} f_i^2 = f_n f_{n+1}$ for all $n \in \mathbf{Z}^+$.

24. $\prod_{i=2}^{n}\{1 - 1/i^2\} = \frac{n+1}{2n}$, where $n \in \mathbf{Z}$ and $n \geq 2$.

Submission Date: 10^{th} November 2015 (Before 1 P.M !)
Odd groups will do Odd questions and Even Groups will solve even questions.

Instructions

1. Solve each question on the sperate sheet.
2. Write formula used in each question.
3. Submit one soft and one hard copy of the solution.
4. No rewards will be given for an assignment submitted after due date.
5. First 11 students on the attendance sheet(ERP) are GROUP#1, next 11 are GROUP#2, next 11 are GROUP#3 and remaining are GROUP#4 (In case of more than 48 students make groups of 12 students,)
6. **An assignment quiz, from the full assignment, will be held in a week's time after the submission date**.