EC-301 Computer Graphics

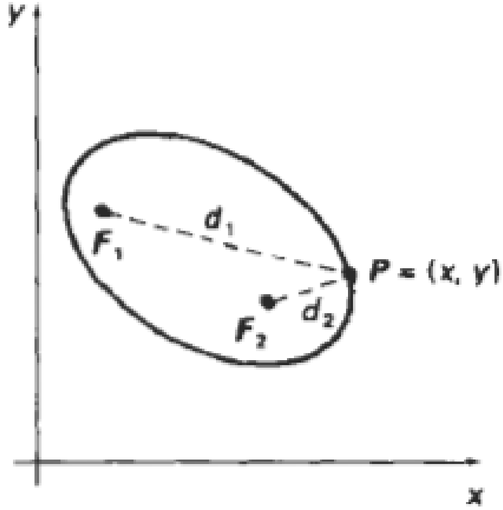
Lecture Slides- Wk12:

- Ellipse-Generating Algorithms
- Midpoint Ellipse Algorithm

Ellipse

An ellipse is a curve that is the locus of all points in the plane, the sum of whose distances d_1 and d_2 from two fixed points F_1 and F_2 (the foci) separated by a distance of 2c is a given positive constant 2a. This results in the two-center bipolar coordinate equation:

$$d_1 + d_2 = 2a$$



Ellipse (Cont.)

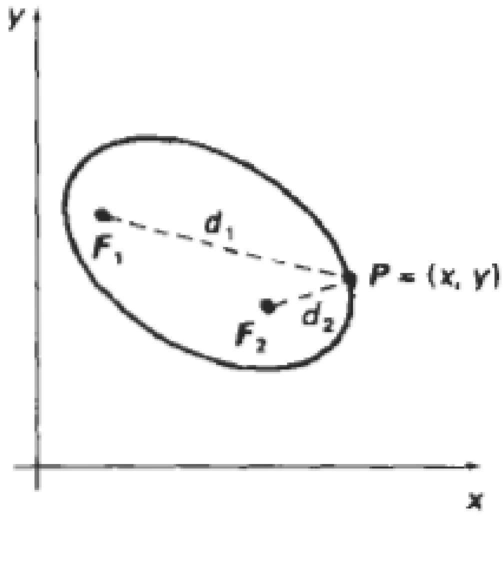
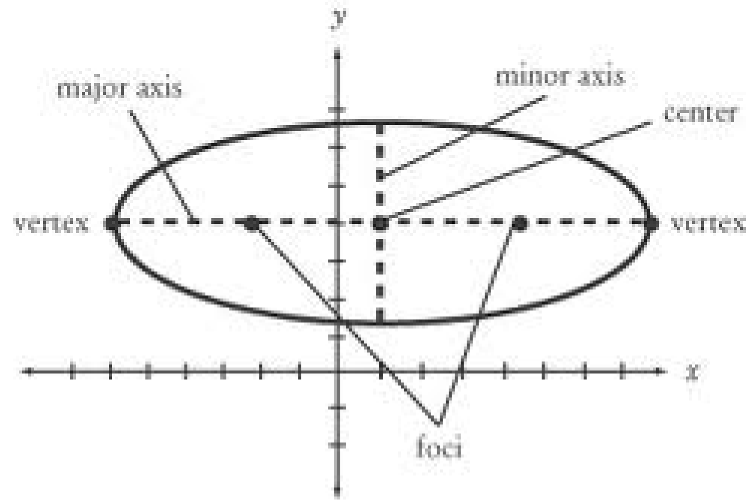


Figure 3-17
Ellipse generated about foci
$\mathbf{F}_1$ and $\mathbf{F}_2$.



$$d_1 + d_2 = \text{constant}$$

$$\sqrt{(x - x_1)^2 + (y - y_1)^2} + \sqrt{(x - x_2)^2 + (y - y_2)^2} = \text{constant}$$

Ellipse (Cont.)

- Let an ellipse lie along the *x*-axis with F1 and F2 being at (-c, 0) and (c, 0).
- In Cartesian coordinates, bring the second term to the right side and square both sides

$$\sqrt{(x+c)^2+y^2}+\sqrt{(x-c)^2+y^2}=2a.$$

$$(x+c)^2+y^2=4a^2-4a\sqrt{(x-c)^2+y^2}+(x-c)^2+y^2.$$

Ellipse (Cont.)

Grouping the x-terms then gives

$$x^2 \frac{a^2 - c^2}{a^2} + y^2 = a^2 - c^2,$$

which can be written in the simple form

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1.$$

defining a new constant

$$b^2 \equiv a^2 - c^2$$

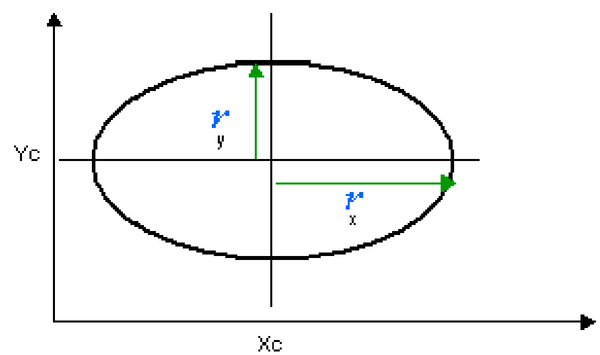
Ellipse (Cont.)

puts the equation in the particularly simple form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

If, instead of being centered at (0, 0), the center of the ellipse is at (x_c, y_c), above equation becomes:

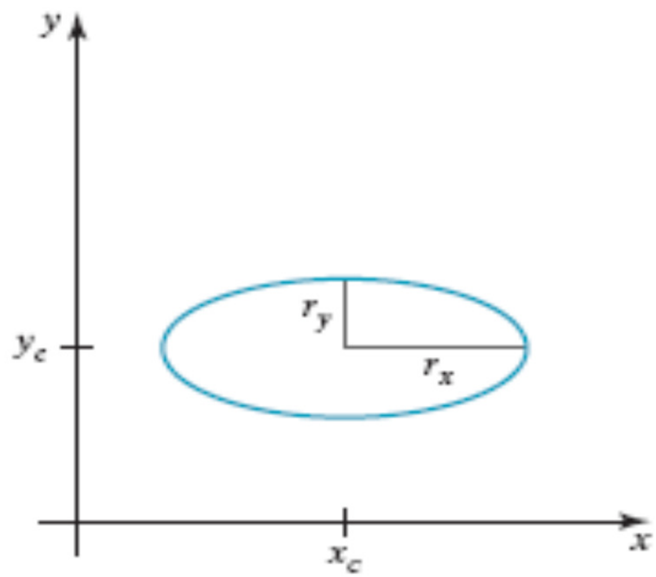
$$\left(\frac{x - x_c}{r_x}\right)^2 + \left(\frac{y - y_c}{r_y}\right)^2 = 1$$



Ellipse-Generating Algorithms

- Ellipse equations are greatly simplified if the major and minor axes are oriented to align with the coordinate axes.
- Figure 3.22 (H&B Book): parameter r_x for this example labels the semi-major axis, and parameter r_y labels the semi-minor axis.

$$\left(\frac{x - x_c}{r_x}\right)^2 + \left(\frac{y - y_c}{r_y}\right)^2 = 1$$



Ellipse-Generating Algorithms

- Input for drawing Ellipse:

  one center point,

  radius r_x and

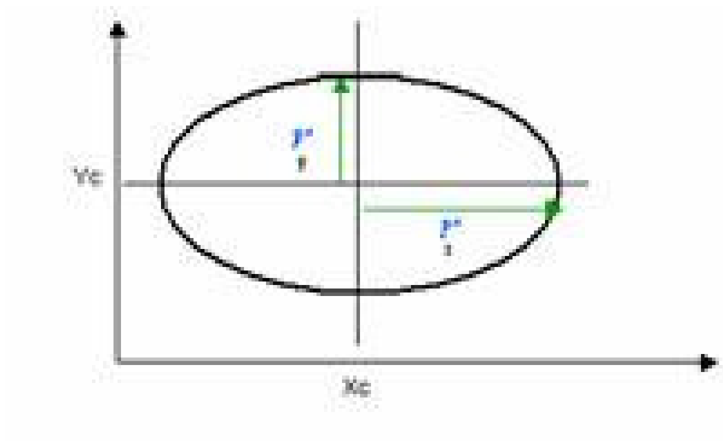
  radius r_y

Now, using these three inputs there are a number of ways to draw an ellipse.

Ellipse-Generating Algorithms

- Keeping in view that you already understand circle drawing techniques, One way to draw ellipse is to use the following equation:

$$\left(\frac{x - x_c}{r_x}\right)^2 + \left(\frac{y - y_c}{r_y}\right)^2 = 1$$

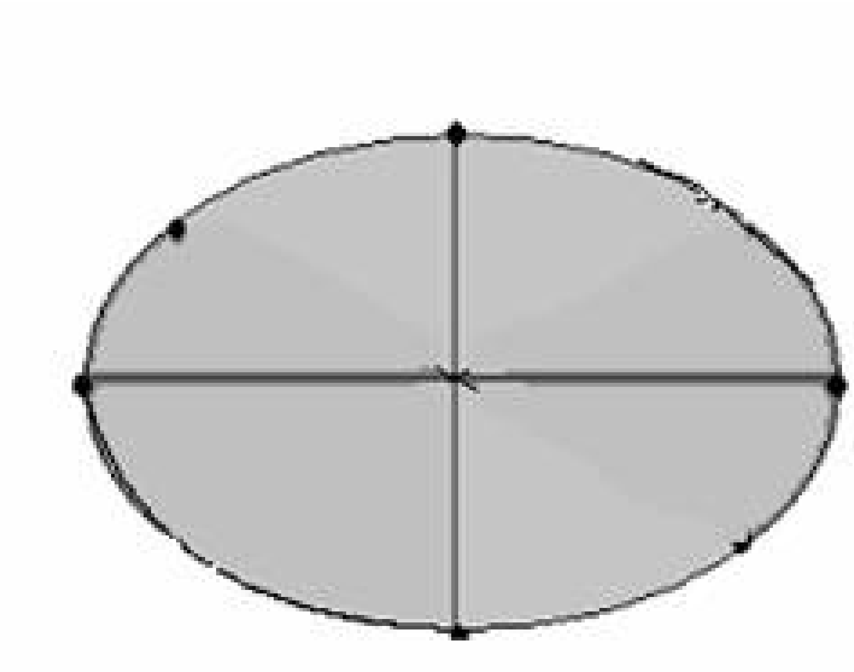


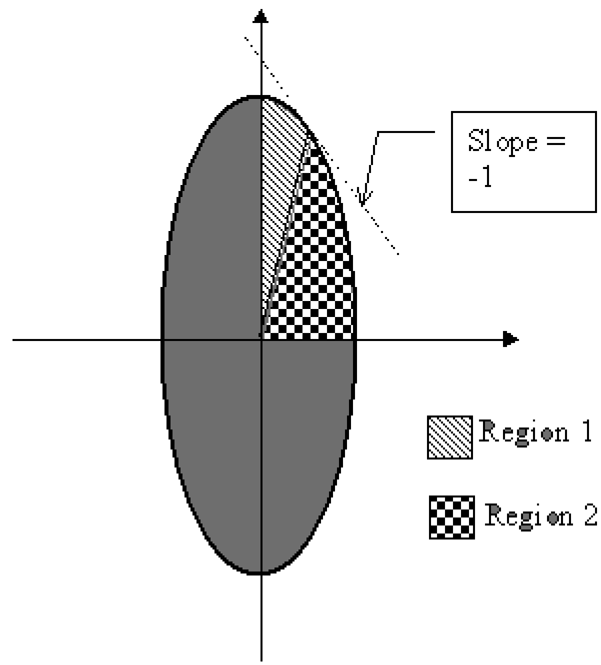
Ellipse-Generating Algorithms

- Another way is to use polar coordinates *r* and θ, for that we have parametric equations:

$$x = x_c + r_x \cos\theta$$
$$y = y_c + r_y \sin\theta \qquad (3\text{-}38)$$

Four-way Symmetry





Midpoint Ellipse Algorithm

- The midpoint ellipse method is applied throughout the first quadrant in two parts.
- Figure 3-25 shows the division of the first quadrant according to the slope of an ellipse with $r_x < r_y$.

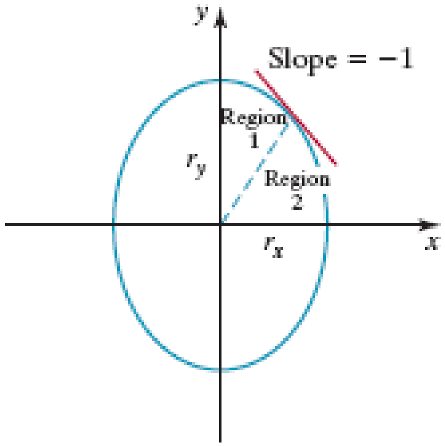


FIGURE 3-25 Ellipse processing regions. Over region 1, the magnitude of the ellipse slope is less than 1.0; over region 2, the magnitude of the slope is greater than 1.0.

Midpoint Ellipse Algorithm

- Regions 1 and 2 (Fig. 3-25) can be processed in various ways.
- We can start at position $(0, r_y)$ and step clockwise along the elliptical path in the first quadrant, shifting from unit steps in x to unit steps in y when the slope becomes less than −1.0.
- Alternatively, we could start at $(r_x, 0)$ and select points in a counterclockwise order, shifting from unit steps in y to unit steps in x when the slope becomes greater than −1.0.

Midpoint Ellipse Algorithm

- We define an ellipse function from Eq. 3-37 (ref: H&B book) with $(x_c, y_c) = (0, 0)$ as

$$f_{\text{ellipse}}(x, y) = r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2 \qquad (3\text{-}39)$$

- which has the following properties:

$$f_{\text{ellipse}}(x, y) \begin{cases} < 0, & \text{if } (x, y) \text{ is inside the ellipse boundary} \\ = 0, & \text{if } (x, y) \text{ is on the ellipse boundary} \\ > 0, & \text{if } (x, y) \text{ is outside the ellipse boundary} \end{cases} \qquad (3\text{-}40)$$