COUNTING

The Addition Principle (AP)
Assume that there are

$$n_1 \text{ ways for the event } E_1 \text{ to occur}$$
$$n_2 \text{ ways for the event } E_2 \text{ to occur}$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$n_k \text{ ways for the event } E_k \text{ to occur} \qquad \text{where } k \geq 1$$

If these ways for the different events to occur are pairwise disjoint, then the number of ways for at least one of the events $E_1, E_2, \cdots, E_k$ to occur

$$n_1 + n_2 + \cdots + n_k = \sum_{i=1}^{k} n_i$$

The Addition Principle (AP) $n_1 + n_2 + \cdots + n_k$

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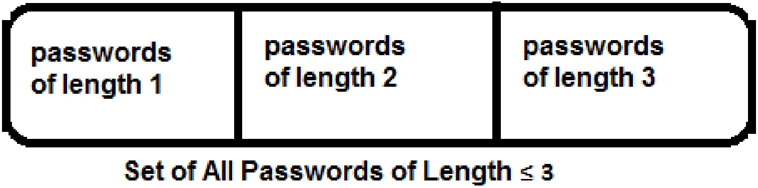

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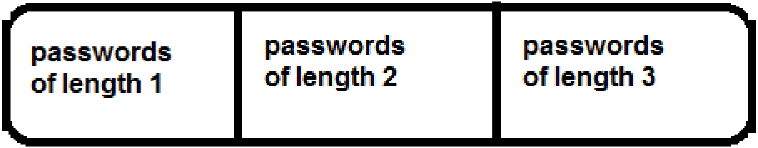

Set of All Passwords of Length ≤ 3

$$\text{number of passwords of length } 1 = 26$$
$$\text{number of passwords of length } 2 = 26^2$$
$$\text{number of passwords of length } 3 = 26^3$$

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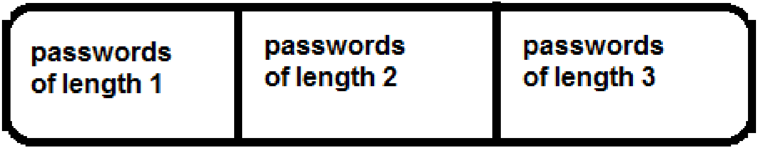

Set of All Passwords of Length ≤ 3

$$\text{number of passwords of length } 1 = 26$$
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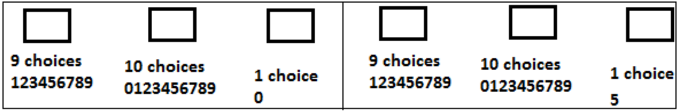
Hence, total number of passwords$= 26 + 26^2 + 26^3 = 18,278$

EXAMPLES

- How many three-digit integers (integers from 100 to 999 inclusive) are divisible by 5?



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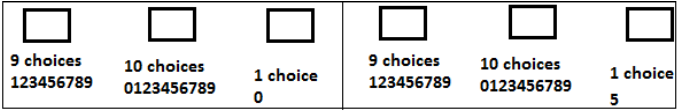

- A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

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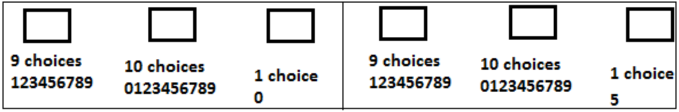

- A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?
- Suppose E is the event of choosing a prime number between 10 and 20, and suppose F is the event of choosing an even number between 10 and 20. Then E can occur in 4 ways $[11, 13, 17, 19]$, and F can occur in 4 ways $[12, 14, 16, 18]$. Then E or F can occur in $4 + 4 = 8$ ways since now none of the even numbers is prime.

The sum rule can be phrased in terms of sets as:
If $A_1, A_2, ..., A_m$ are pairwise disjoint finite sets, then the number of elements in the union of these sets is the sum of the numbers of elements in the sets.

$$|A_1 \cup A_2 \cup \cdots \cup A_m| = |A_1| + |A_2| + \cdots + |A_m|$$

The Multiplication Principle (MP)

Assume that an event E can be decomposed into r ordered events $E_1, E_2, ..., E_r$, and that there are

n_1 ways for the event E_1 to occur

n_2 ways for the event E_2 to occur

$$\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots$$

n_r ways for the event E_r to occur where $r \geq 1$

Then the total number of ways for the event E to occur is given by:

$$n_1 \times n_2 \times \cdots \times n_r = \prod_{i=1}^{r} n_i$$

1. The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?
2. There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a microcomputer in the center are there?
3. How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)?
4. Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit[1]. How many possible passwords are there?

[1] number of strings of uppercase letters and digits that are six characters long, including those with no digits, and subtract from this the number of strings with no digits.

1. There are 18 mathematics majors and 325 computer science majors at a college.
   - (a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
   - (b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

[2] (a) 5850, (b) 343

1. There are 18 mathematics majors and 325 computer science majors at a college.
   - (a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
   - (b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?[2]
2. A multiple-choice test contains 10 questions. There are four possible answers for each question.
   - (a) In how many ways can a student answer the questions on the test if the student answers every question?
   - (b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

[2] (a) 5850, (b) 343

3. A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?
4. How many positive integers between 100 and 999 inclusive
   - *(i)* are divisible by 7?
   - *(ii)* are odd?
   - *(iii)* have the same three decimal digits?
   - *(iv)* are not divisible by 4?
   - *(v)* are divisible by 3 or 4?
   - *(vi)* are not divisible by either 3 or 4?
   - *(vii)* are divisible by 3 but not by 4?
   - *(viii)* are divisible by 3

The Principle of Inclusion and Exclusion

If A and B are finite sets such that $A \cap B = 0$, then $|A \cup B| = |A| + |B|$.

What is the corresponding equality for $|A \cup B|$ if $A \cap B \neq \emptyset$?

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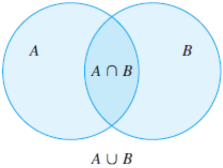

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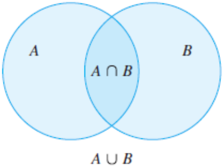

$$|A \cup B| = |A| + |B| - |A \cap B|$$

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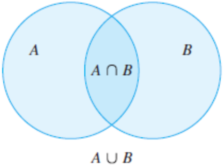
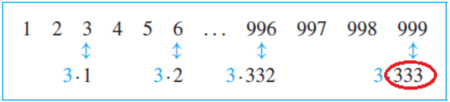


$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - \{|A \cap B| + |B \cap C| + |A \cap C|\} + |A \cap B \cap C|$$

How many integers from 1 through 1,000 are multiples of 3 or multiples of 5?



A = the set of all integers from 1 through 1,000 that are multiples of 3.

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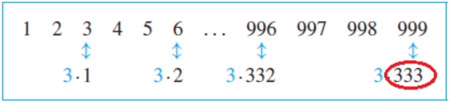

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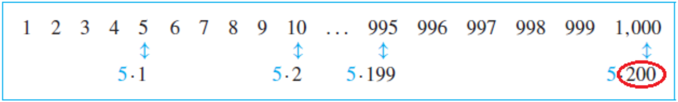

B = the set of all integers from 1 through 1,000 that are multiples of 5.

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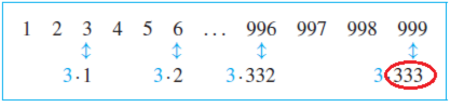

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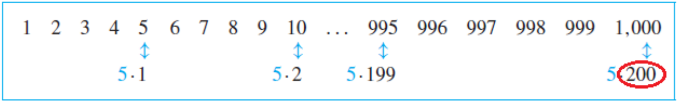

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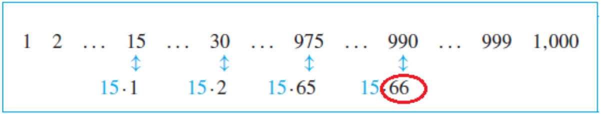

$A \cap B$ = the set of all integers from 1 through 1,000 that are multiples of both 3 and 5
= the set of all integers from 1 through 1,000 that are multiples of 15.

How many integers from 1 through 1,000 are multiples of 3 or multiples of 5?

1	2	3	4	5	6	...	996	997	998	999
		↕			↕		↕			↕
		$3 \cdot 1$			$3 \cdot 2$		$3 \cdot 332$			$3 \cdot 333$

$A =$ the set of all integers from 1 through 1,000 that are multiples of 3.

1	2	3	4	5	6	7	8	9	10	...	995	996	997	998	999	1,000
				↕					↕		↕					↕
				$5 \cdot 1$					$5 \cdot 2$		$5 \cdot 199$					$5 \cdot 200$

$B =$ the set of all integers from 1 through 1,000 that are multiples of 5.

1	2	...	15	...	30	...	975	...	990	...	999	1,000
			↕		↕		↕		↕			
			$15 \cdot 1$		$15 \cdot 2$		$15 \cdot 65$		$15 \cdot 66$			

$A \cap B =$ the set of all integers from 1 through 1,000 that are multiples of 15.

$A \cup B =$ the set of all integers from 1 through 1,000 that are multiples of 3 or multiples of 5

How many integers from 1 through 1,000 are multiples of 3 or multiples of 5?

1 2 3 4 5 6 . . . 996 997 998 999

$3 \cdot 1$ (↔ 3), $3 \cdot 2$ (↔ 6), $3 \cdot 332$ (↔ 996), $3 \cdot 333$ (↔ 999)

$A =$ the set of all integers from 1 through 1,000 that are multiples of 3.

1 2 3 4 5 6 7 8 9 10 . . . 995 996 997 998 999 1,000

$5 \cdot 1$ (↔ 5), $5 \cdot 2$ (↔ 10), $5 \cdot 199$ (↔ 995), $5 \cdot 200$ (↔ 1,000)

$B =$ the set of all integers from 1 through 1,000 that are multiples of 5.

1 2 . . . 15 . . . 30 . . . 975 . . . 990 . . . 999 1,000

$15 \cdot 1$ (↔ 15), $15 \cdot 2$ (↔ 30), $15 \cdot 65$ (↔ 975), $15 \cdot 66$ (↔ 990)

$A \cap B =$ the set of all integers from 1 through 1,000 that are multiples of 15.

$A \cup B =$ the set of all integers from 1 through 1,000 that are multiples of 3 or multiples of 5

$$|A \cup B| = |A| + |B| - |A \cap B| = 333 + 200 - 66 = ?$$

How many integers from 1 through 1,000 are ***neither*** *multiples of 3* ***nor*** *multiples of 5?*

The number of elements that are neither in A nor in B is $|A^c \cap B^c|$,

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The number of elements that are neither in A nor in B is $|A^c \cap B^c|$,
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$$|A^c \cap B^c| = |(A \cup B)^c| = |U| - |A \cup B|$$

where the universe U was the set of all integers from 1 through 1,000.

$$|A \cup B| = |A| + |B| - |A \cap B| = 333 + 200 - 66 = 467$$

$$|A^c \cap B^c| = |(A \cup B)^c| = |U| - |A \cup B| = 1000 - 467 = 533$$

Example

A computer company receives 350 applications from computer graduates for a job planning a line of new Web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?

Let A_1 be the set of students who majored in computer science

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$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2| = 220 + 147 - 51 = ?$$