



Subject Code : Math-161	Subject : Discrete Mathematics
Date : 29-01-2015	Time allowed : 3 hrs
Max. Marks : 100	Instructor : Dr. Yasir Ali
Note: Attempt all questions. All parts of Question:1 carry 5 marks each.	

Question:1	<p>Give brief answers to the following questions</p> <ol style="list-style-type: none"> 1. What is the maximum number of edges a simple disconnected graph with n vertices can have? Prove your answer. 2. For what values of m and n does the complete bipartite graph on (m, n) vertices have an Euler circuit? 3. What is the total degree of a tree with n vertices? Why? 4. Either draw a graph with the given specifications or explain why no such graph exists: <ol style="list-style-type: none"> (i) Tree, nine vertices, nine edges (ii) Graph, connected, nine vertices, nine edges 5. If a graph has n vertices and $n - 2$ or fewer edges, can it be connected? Why? 6. Prove that if seven distinct numbers are selected from $\{1, 2, \dots, 11\}$, then some two of these numbers sum to 12. 7. Let $E = \{a, b, c, \dots, x, y, z\}$ be the set of the 26 English alphabets. Find the number of 5-letter words that can be formed from E such that the first and last letters are distinct vowels and the remaining three are distinct consonants. 8. Assume that R and S are relations on a set A. Prove or disprove that if R and S are reflexive, is $R \cap S$ reflexive? Why? 	(40)
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<p>Question:2</p> <p>(a)</p>	<p>Prove or disprove using mathematical induction</p> <p>(i) $\sum_{i=0}^n ar^i = \frac{ar^{n+1}+a}{r+1}$</p> <p>(ii) $\sum_{i=0}^n 2^i = 2^{n+1} - 1$</p> <p>(b) Solve the recurrence relation $a_n - 3a_{n-1} + 2a_{n-2} = 2^n$ given that $a_0 = 3$ and $a_1 = 8$.</p>	<p>(05)</p> <p>(10)</p>
<p>Question:3</p> <p>(a)</p> <p>(b)</p>	<p>There are 50 baskets of apples. Each basket contains no more than 24 apples. Find the minimum number of baskets containing the same number of apples, by using Pigeonhole Principle.</p> <p>How many bit strings of length eight contain either three consecutive 0s or four consecutive 1s? (Use Principle of Inclusion and exclusion)</p>	<p>(05)</p> <p>(10)</p>
<p>Question:4</p> <p>(a)</p> <p>(b)</p>	<p>Analyze function $3x^2 + 8x \log x$ to show that the is $\Theta(x^2)$.</p> <p>Infer that $1 + 2 + 2^2 + 2^3 + \dots + 2^n$ is $\Theta(n)$. Assuming n is an integer variable that takes positive integer values.</p>	<p>(05)</p> <p>(10)</p>
<p>Question:5</p> <p>(a)</p> <p>(b)</p>	<p>In graphs given above (fig-(i) and fig-(ii)), find</p> <p>(i) parallel edges, (ii) Euler'circuites, (iii) Adjancecy list</p> <p>Show the steps in the execution of Dijkstras shortest path algorithm for the graph shown below with starting vertex a to all other vertices. Maintain a lists of Predecessors and Shortest Path estimates for all vertices.</p>	<p>(05)</p> <p>(10)</p>