Lecture 2

EE-215 Electronic Devices and Circuits

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Modeling the Diode Forward Characteristic

- The Exponential Model
- The Constant-Voltage Drop Model
- The Ideal Diode Model
- The Small-Signal Model

The Exponential Model

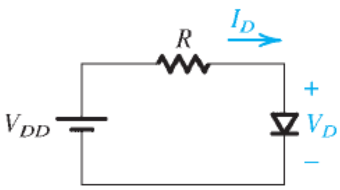
- provides the most accurate description of diode operation
    - in the forward-bias region
- is the most difficult to use
    - because of its severely non-linear nature
- Exponential model $= \Rightarrow i = I_S e^{\frac{v}{V_T}}$
- 

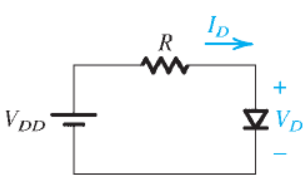
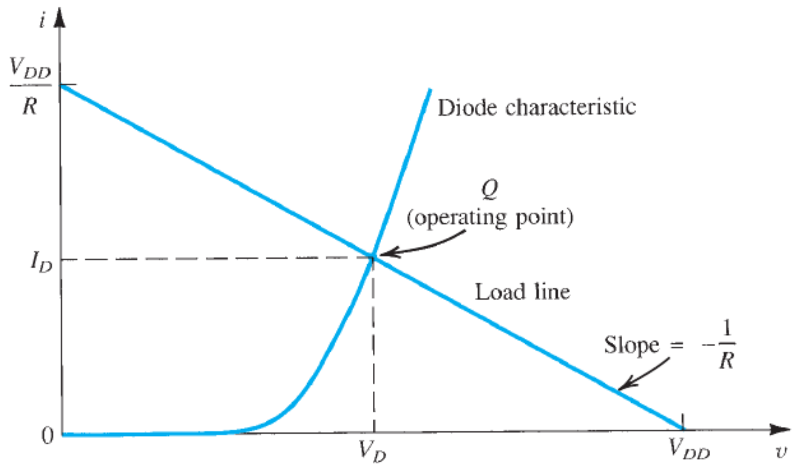
Figure 4.10 A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting.
    - for this circuit
        - $I_D = I_S e^{\frac{V_D}{V_T}}$
    - and using KVL
        - $V_{DD} = I_D R + V_D$
        - or $I_D = \frac{V_{DD} - V_D}{R}$
    - if I_S for the diode is given,
        - we have two equations in two unknowns I_D and V_D
- $I_D = I_S e^{\frac{V_D}{V_T}}$ and $I_D = \frac{V_{DD} - V_D}{R}$
    - 

Figure 4.10 A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting.
- if I_S for the diode is given,
    - we have two equations in two unknowns I_D and V_D

- These two equations can be solved using either
  - Graphical Analysis
  - or Iterative Analysis

Graphical Analysis using the Exponential Model

- $I_D = I_S e^{\frac{V_D}{V_T}}$ and $I_D = \frac{V_{DD} - V_D}{R}$
- this analysis is performed by
  - plotting the two equations on the i-v plane
  - the point of intersection is the desired solution (V_D, I_D)

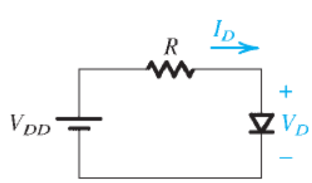


  - **Figure 4.11** Graphical analysis of the circuit in Fig. 4.10 using the exponential diode model.

- the curve represents the exponential diode equation.
  - while the straight line represents the KVL equation
    - Such a straight line is called load line.
- The load line intersects the diode curve at point Q
  - Q represents the operating point of the circuit.
  - The coordinates of Q give the values of I_D and V_D

Example 4.4 using Graphical Analysis

- Determine the current I_D and the diode voltage V_D for the circuit in Fig. 4.10 with $V_{DD} = 5V$ and $R = 1k\Omega$. Assume that the diode has a current of $1mA$ at a voltage of $0.7V$.

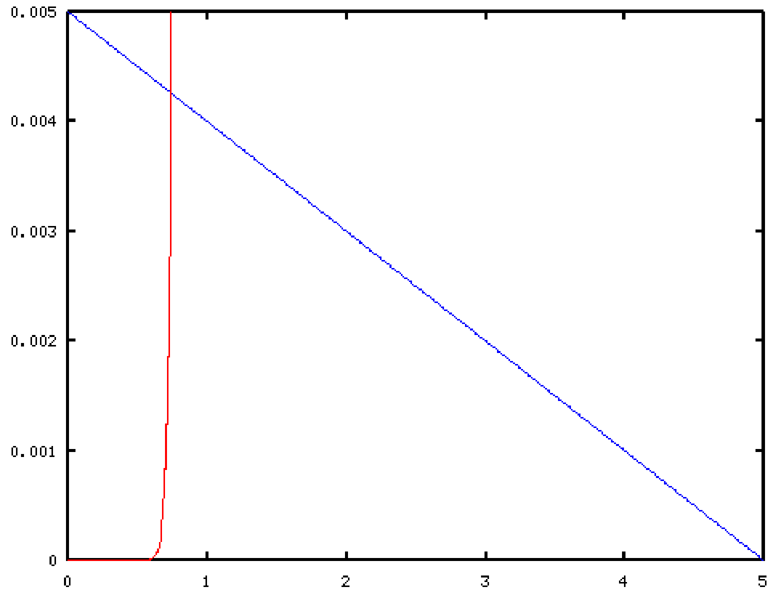


  - **Figure 4.10** A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting.

Solution:

- here $V_{DD} = 5V$, $R = 1k\Omega$
  - and $I = 1mA$ for $V = 0.7V$
    - $I = I_S e^{\frac{V}{V_T}} = \Rightarrow I_S = \frac{I}{e^{\frac{V}{V_T}}} = \frac{1mA}{e^{\frac{0.7V}{25mV}}} = 6.9144 \times 10^{-16} A$

- As the 2 circuit equations are $I_D = I_S e^{\frac{V_D}{V_T}}$ and $I_D = \frac{V_{DD} - V_D}{R}$
- $= \Rightarrow$ $I_D = 6.9144 \times 10^{-16} e^{\frac{V_D}{25m}}$ and $I_D = \frac{5 - V_D}{1k}$

* $I_D = 6.9144 \times 10^{-16} e^{\frac{V_D}{25m}}$ and $I_D = \frac{5 - V_D}{1k}$
    - Plotting these 2 equations in Matlab or Octave with give the desired solution V_D, I_D
    - 

* reading V_D, I_D from the plot $= \Rightarrow$ $V_D = 0.7358V$
    - and $I_D = 4.264mA$

Iterative Analysis using the Exponential Model

* the 2 circuit equations
    - $I_D = I_S e^{\frac{V_D}{V_T}}$ and $I_D = \frac{V_{DD} - V_D}{R}$
    - can also be solved using a simple iterative procedure

Modeling the Diode Forward Characteristic

* $I_D = I_S e^{\frac{V_D}{V_T}}$ and $I_D = \frac{V_{DD} - V_D}{R}$

Example 4.4 using Iterative Analysis

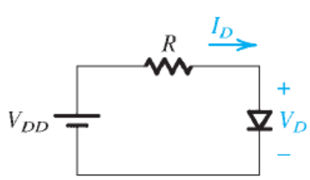
* Determine the current I_D and the diode voltage V_D for the circuit in Fig. 4.10 with $V_{DD} = 5V$ and $R = 1k\Omega$. Assume that the diode has a current of $1mA$ at a voltage of $0.7V$.
    - 

Figure 4.10 A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting.

Solution:

- to start, let $V_D = 0.7V$

Iteration 1:

- KVL $= \Rightarrow I_D = \frac{V_{DD} - V_D}{R} = \frac{5 - 0.7}{1k} = 4.3mA$
  - to have a better estimate of V_D
    - using $V_2 - V_1 = V_T \ln \frac{I_2}{I_1}$ or $V_2 = V_1 + V_T \ln \frac{I_2}{I_1}$
    - for $V_1 = 0.7V$ and $I_1 = 1mA$ (given)
    - and $V_2 = V_D = ?$, $I_2 = I_D = 4.3mA$
    - $V_2 = V_1 + V_T \ln \frac{I_2}{I_1} = \Rightarrow V_D = 0.7 + 25m \ln \frac{4.3m}{1m} = 0.736V$
  - thus after this first iteration we have
    - $V_D = 0.736V$, $I_D = 4.3mA$
- Iteration1 $= \Rightarrow V_D = 0.736V$, $I_D = 4.3mA$

Iteration 2:

- for the 2nd iteration
  - $I_D = \frac{V_{DD} - V_D}{R} = \frac{5 - 0.736}{1k} = 4.264mA$
    - $V_2 = V_1 + V_T \ln \frac{I_2}{I_1}$
    - here $V_1 = V_D$ from first iteration, $I_1 = I_D$ from first iteration
  - $= \Rightarrow V_D = 0.736 + 25m \ln \frac{4.264m}{4.3m} = 0.7358V \approx 0.736V$
    - so 2nd iteration $= \Rightarrow V_D = 0.736V$ and $I_D = 4.264mA$
  - As the values obtained in the 2nd iteration
    - are very close to those obtained in the first iteration,
    - further iterations are not required
  - so the solution is $V_D = 0.736V$, $I_D = 4.264mA$

Example 4.4 using Graphical and Iterative Analysis

- Iterative Analysis solution : $V_D = 0.736V$, $I_D = 4.264mA$
  - Note that this solution closely match (exactly match to the 4 decimal),
  - to the one obtained using Graphical Analysis
  - i.e. $V_D = 0.7358V$, $I_D = 4.264mA$ (Graphical Analysis solution)

The Constant Voltage Drop Model

- is the simplest
  - and most widely used model
  - is based on the observation that a conducting diode has
  - voltage drop in a narrow range of 0.6V - 0.8V

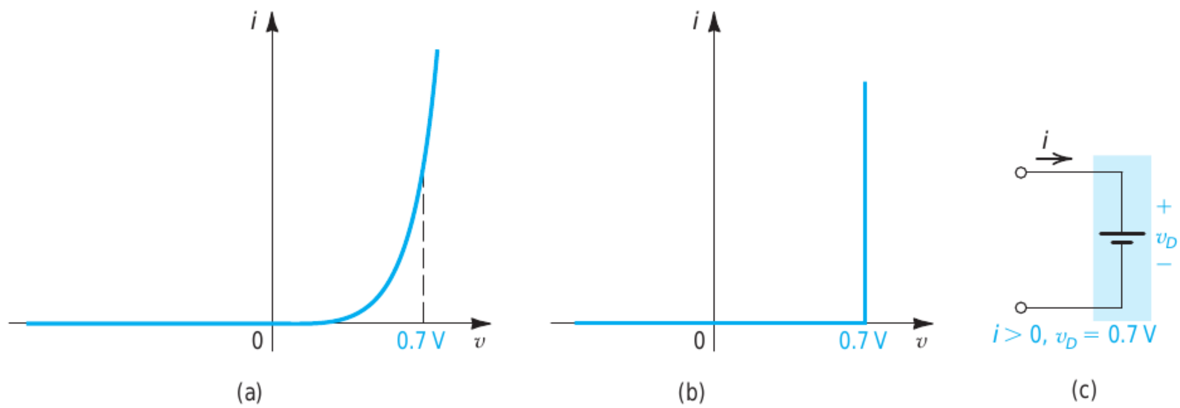
- thus the model assumes a constant value of 0.7V
- 

Figure 4.12 Development of the diode constant-voltage-drop model: **(a)** the exponential characteristic; **(b)** approximating the exponential characteristic by a constant voltage, usually about 0.7 V **(c)** the resulting model of the foward–conducting diodes.

Example 4.4 using The Constant-Voltage-Drop Model

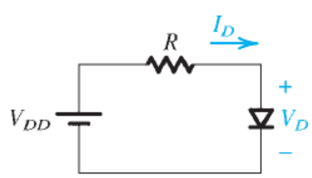
- Determine the current I_D and the diode voltage V_D for the circuit in Fig. 4.10 with $V_{DD} = 5V$ and $R = 1k\Omega$. Assume that the diode has a current of $1mA$ at a voltage of $0.7V$.
    - 

Figure 4.10 A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting.

Solution:

- constant voltage drop model $= \Rightarrow V_D = 0.7V$
    - KVL $= \Rightarrow I_D = \frac{V_{DD}-V_D}{R} = \frac{5-0.7}{1k} = 4.3mA$
    - Graphical Analysis solution $= \Rightarrow V_D = 0.7358V, I_D = 4.264mA$

The Ideal-Diode Model

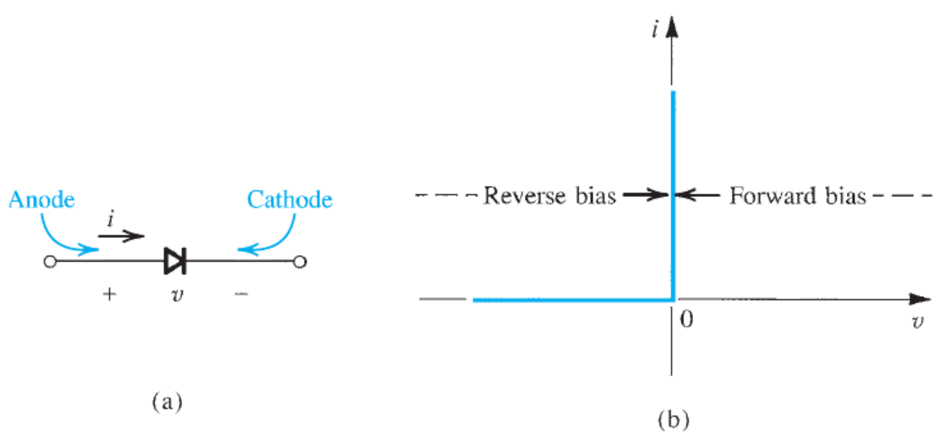
- Already studied in detail
    - 

Figure 4.1 The ideal diode: **(a)** diode circuit symbol; **(b)** i–v characteristic;
    - The most commonly used model
        - is the constant-voltage-drop model

- The ideal diode model is very useful in determining
  - which diodes are on and which are off
  - in a multi-diode circuit

Example 4.4 using The Ideal-diode Model

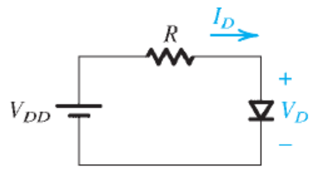
- Determine the current I_D and the diode voltage V_D for the circuit in Fig. 4.10 with $V_{DD} = 5V$ and $R = 1k\Omega$. Assume that the diode has a current of $1mA$ at a voltage of $0.7V$.
  - 

Figure 4.10 A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting.

Solution:

- ideal diode model $= \Rightarrow V_D = 0V$
  - KVL $= \Rightarrow I_D = \frac{V_{DD} - V_D}{R} = \frac{5 - 0}{1k} = 5mA$

Modeling the Diode Forward Characteristic

Example 4.4

- Exponential Model using Graphical Analysis$= \Rightarrow V_D = 0.7358V, I_D = 4.264mA$
- Exponential Model using Iterative Analysis$= \Rightarrow V_D = 0.736V, I_D = 4.264mA$
- Constant-Voltage-Drop model$= \Rightarrow V_D = 0.7V, I_D = 4.3mA$
- Ideal-Diode-Model$= \Rightarrow V_D = 0V, I_D = 5mA$

Exercise D4.11

- Design the circuit in Fig E4.11 to provide an output voltage of 2.4V. Assume that the diodes available have 0.7V drop at 1mA.

Solution:

- here $V_1 = 0.7V, I_1 = 1mA$
  - required is $V_O = 2.4V$
    - $= \Rightarrow V_D = \frac{V_O}{3} = \frac{2.4}{3} = 0.8V$
      - $I_D = ?$
        - $V_2 - V_1 = V_T \ln \frac{I_2}{I_1}$
          - here $V_2 = 0.8V$ and $I_D = I_2 = ?$
          - $V_2 - V_1 = V_T \ln \frac{I_2}{I_1} = \Rightarrow 0.8 - 0.7 = V_T \ln \frac{I_D}{1m}$
          - or $0.1 = V_T \ln \frac{I_D}{1m} = \Rightarrow \frac{0.1}{25m} = \ln \frac{I_D}{1m}$

- or $\frac{I_D}{1m} = e^{\frac{0.1}{25m}} = \Rightarrow I_D = 1me^{\frac{0.1}{25m}} = 54.6mA$

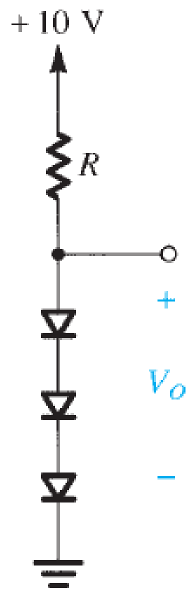


Figure E4.11

- $V_O = 2.4V$
  - $I_D = 54.6mA$
  - AsI_D is the current flowing through resistor R
    - Applying ohm's law at R
      - $= \Rightarrow I_D = \frac{10 - 2.4}{R} = 54.6mA$
      - or $R = \frac{10 - 2.4}{54.6m} = \frac{7.6}{54.6m} = 139.19\Omega$

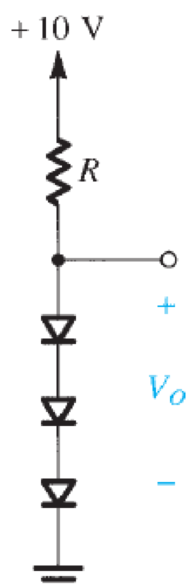
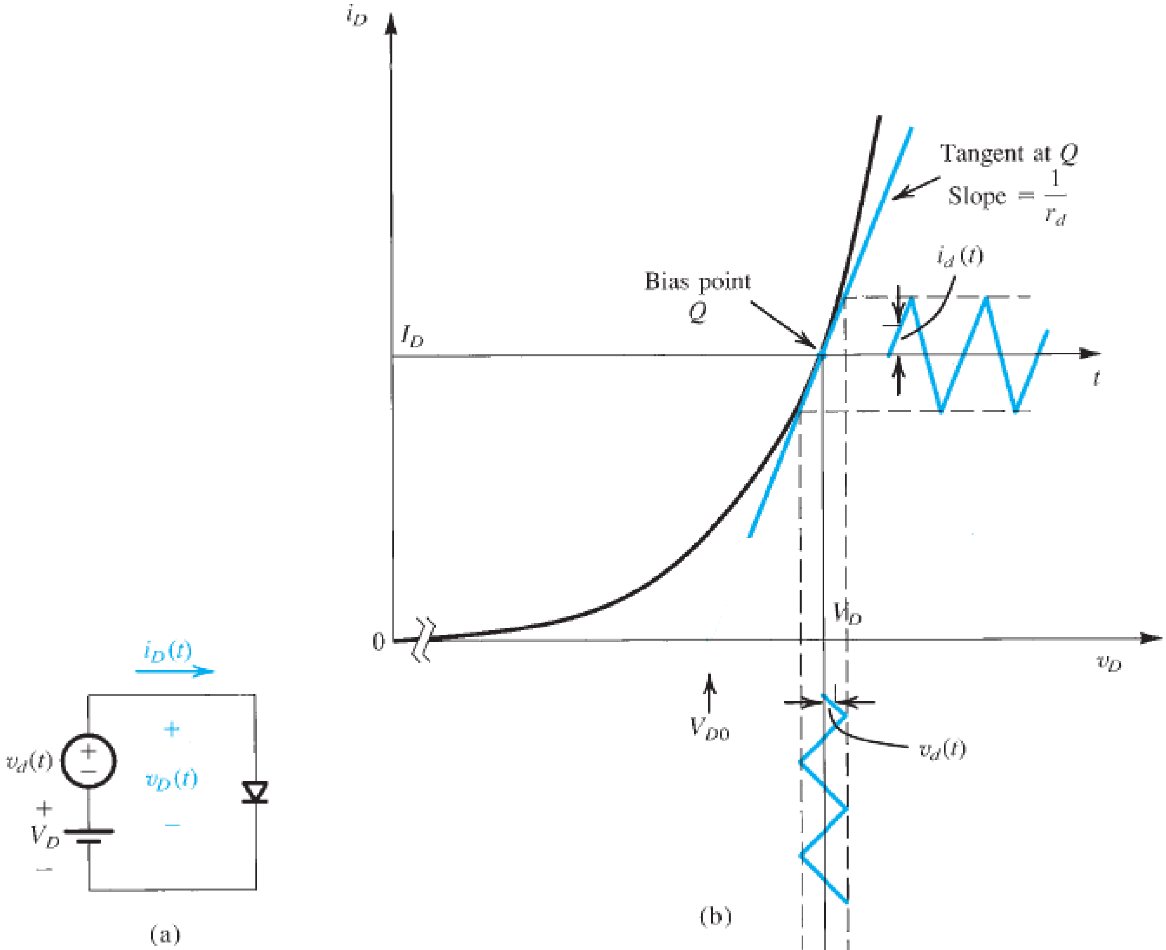


Figure E4.11

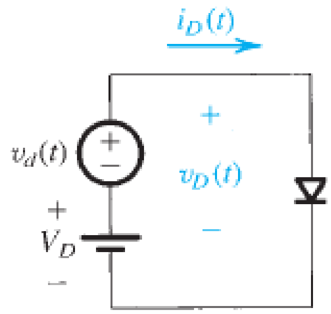
The Small-Signal Model

- this model is useful in applications where
  - a diode is forward biased

- and a small AC signal is superimposed on the DC quantities
- in such applications,
    - one can first determine the DC operating point (V_D , I_D) of the diode
    - then for small signal operation, around the DC bias point,
    - the diode is modeled by a resistance equal to the inverse of the slope of the tangent to the exponential i-v characteristic at the bias point.



- **Figure 4.13** Development of the diode small-signal model.
- here a dc voltage V_D, represented by a battery is applied to the diode
    - and a time-varying signal $v_d\left(t\right)$, is superimposed on the dc voltage V_D
        - In the absence of $v_d\left(t\right)$, the diode voltage equals V_D
        - and the diode current is $I_D = I_S e^{V_D / V_T}$

- 

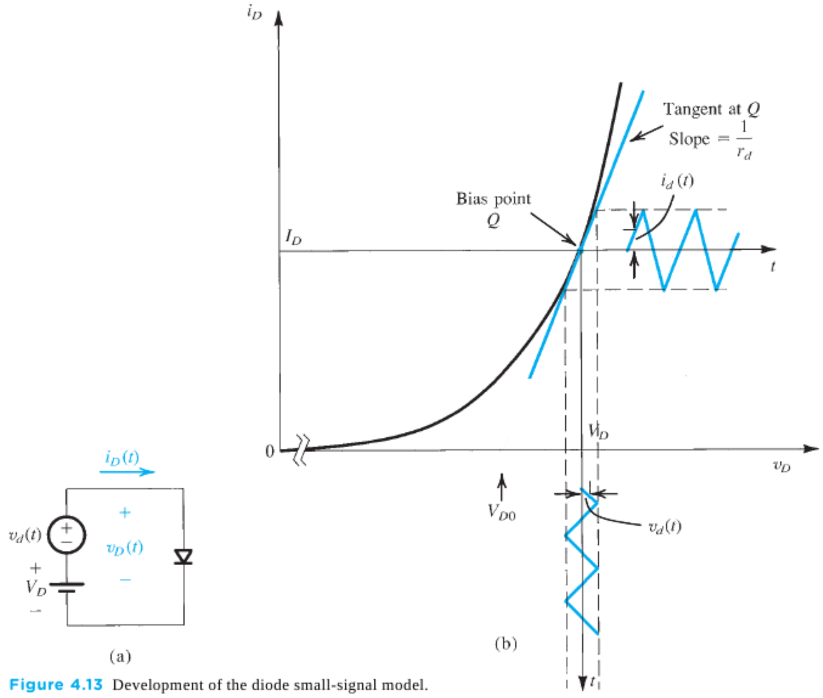
(a)
Figure 4.13 Development of the diode small-signal model.
  - when the signal $v_d(t)$ is applied, the total instantaneous diode voltage $v_D(t)$ is
    - $v_D(t) = V_D + v_d(t)$
  - and the total current is $= \Rightarrow i_D(t) = I_S e^{v_D(t)/V_T}$
- $i_D(t) = I_S e^{v_D(t)/V_T}$
  - $i_D(t) = I_S e^{(V_D + v_d)/V_T} \because v_D(t) = V_D + v_d(t)$
    - $i_D(t) = I_S e^{V_D/V_T} e^{v_d/V_T}$
    - $i_D(t) = I_D e^{v_d/V_T} \because I_D = I_S e^{V_D/V_T}$
- As in terms of series, exponential can be represented as
  - $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$
  - $= \Rightarrow i_D\left(t\right) = I_D e^{v_d/V_T} = I_D\left\{1 + \frac{v_d}{V_T} + \frac{1}{2!}\left(\frac{v_d}{V_T}\right)^2 + \cdots\right\}$
- $i_D\left(t\right) = I_D\left\{1 + \frac{v_d}{V_T} + \frac{1}{2!}\left(\frac{v_d}{V_T}\right)^2 + \cdots\right\}$
  - now if the amplitude of the signal $v_d(t)$ is kept sufficiently small i.e. $\frac{v_d}{V_T} < 1 = \Rightarrow$

$\left(\frac{v_d}{V_T}\right)^2 < \frac{v_d}{V_T}$

- $= \Rightarrow \left\{1 + \frac{v_d}{V_T} + \frac{1}{2!}\left(\frac{v_d}{V_T}\right)^2 + \cdots\right\} \approx \left\{1 + \frac{v_d}{V_T}\right\}$

- $= \Rightarrow i_D\left(t\right) = I_D\left\{1 + \frac{v_d}{V_T} + \frac{1}{2!}\left(\frac{v_d}{V_T}\right)^2 + \cdots\right\} \approx I_D\left\{1 + \frac{v_d}{V_T}\right\}$
- this is the small signal approximation.
  - and this approximation is valid for signals whose amplitudes are smaller than about 5mV
  - e.g. if $v_d = 5mV = \Rightarrow \frac{v_d}{V_T} = \frac{5m}{25m} = 0.2$
    - 3rd term in series=$\frac{1}{2!}\left(\frac{v_d}{V_T}\right)^2 = \frac{1}{2!}(0.2)^2 = 0.02$ which is 10 times smaller than 2nd term and can be ignored.

- $i_D\left(t\right) \approx I_D\left\{1 + \frac{v_d}{V_T}\right\}$
  - or $i_D\left(t\right) = I_D + \frac{I_D}{V_T}v_d$
  - thus we have a signal (ac) current $\left(i_d = \frac{I_D}{V_T}v_d\right)$ superimposed on the dc current I_D
  - i.e. $i_D = I_D + i_d$
    - where $i_d = \frac{I_D}{V_T}v_d$
    - thus the signal current i_d is directly proportional to the signal voltage v_d
  - this quantity relating the signal current i_d to the signal voltage v_d is called the small-signal conductance.
    - small-signal conductance $= \frac{i_d}{v_d} = \frac{I_D}{V_T}$
  - the inverse of this conductance is the diode small-signal resistance (also called diode incremental resistance)
- diode small-signal resistance $= r_d = \frac{V_T}{I_D} = \frac{v_d}{i_d}$
  - thus r_d is inversely proportional to the diode bias current I_D
  - One can see the small signal approximation from the graphical representation



- Figure 4.13 Development of the diode small-signal model.

- Using the small-signal approximation is equivalent to
  - assuming that the signal amplitude is sufficiently small such that
    - the signal swing along the i-v curve is limited to a short almost-linear segment
  - the slope of this linear segment
    - equals the slope of the tangent to the i-v curve at the operating point Q
    - equals the small signal conductance
  - $= \Rightarrow \frac{1}{r_d} = \left[\frac{\partial i_D}{\partial v_D}\right]_{i_D = I_D}$
    - i.e. slope of the i-v curve at $i = I_D$ is equal to $\frac{1}{r_d} = \frac{I_D}{V_T}$
- $\frac{1}{r_d} = \left[\frac{\partial i_D}{\partial v_D}\right]_{i_D = I_D}$
  - $\frac{\partial i_D}{\partial v_D} = \frac{\partial}{\partial v_D}\left(I_S e^{v_D / V_T}\right) \because i_D = I_S e^{v_D / V_T}$
    - $\frac{\partial i_D}{\partial v_D} = I_S e^{v_D / V_T} \frac{\partial}{\partial v_D}\left(v_D / V_T\right) = I_S e^{v_D / V_T}\left(\frac{1}{V_T}\right)$
  - As $i_D = I_S e^{v_D / V_T}$

            - $\frac{\partial i_D}{\partial v_D} = I_S e^{v_D / V_T}\left(\frac{1}{V_T}\right) = \frac{i_D}{V_T}$

    - $= \Rightarrow \frac{1}{r_d} = \left[\frac{\partial i_D}{\partial v_D}\right]_{i_D = I_D} = \left[\frac{i_D}{V_T}\right]_{i_D = I_D} = \frac{I_D}{V_T}$

- The small signal model
    - $= \Rightarrow$ dc and ac analysis can be performed independently
    - first dc analysis is carried out, which leads to the Q point
        - then the small signal equivalent circuit is obtained by
        - eliminating all dc sources
    - i.e. replacing dc voltage source by short circuit
        - replacing dc current source by open circuit
        - and replacing the diode by its small-signal resistance $r_d = \frac{V_T}{I_D}$

Example 4.5

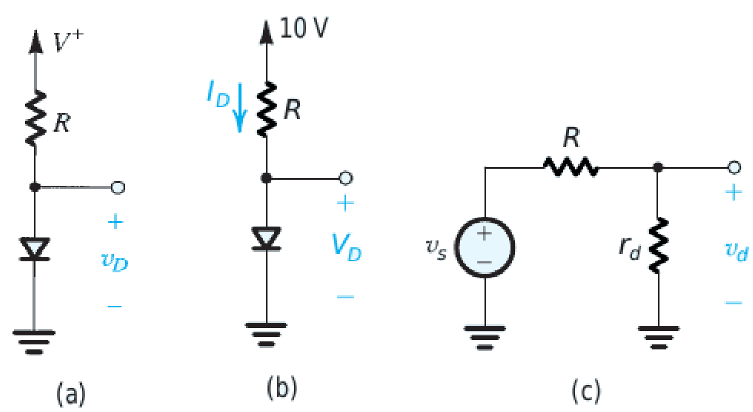
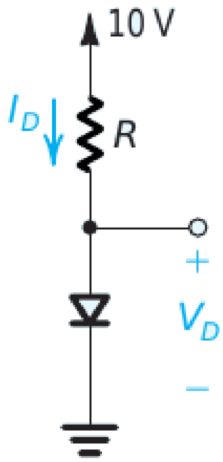
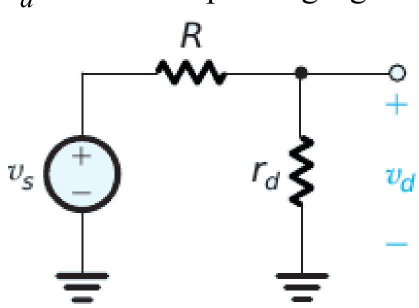
- Consider the circuit shown in Fig 4.14(a) for the case in which $R = 10k\Omega$. The power supply V^+ has a dc value of 10 V on which is superimposed a 60-Hz sinusoid of 1-V peak amplitude. (This "signal component" of the power suply voltage is an imperfection in the power-supply design. It is known as the power-supply ripple.) Calculate the dc voltage of the diode and the amplitude of the sine-wave signal appearing across it. Assume the diode to have a 0.7-V drop at 1 mA current.
    - 

Figure 4.14
(a) Circuit for Example 4.5.
(b) Circuit for calculating the dc operating point.
(c) Small-signal equivalent circuit.

- using the small signal model $= \Rightarrow$ dc analysis can be performed first followed by small-signal analysis
- dc analysis
    - let $V_D \approx 0.7V$
    - ohm's law across R $= \Rightarrow$
        - $I_D = \frac{10 - 0.7}{R} = \frac{9.3}{10k} = 0.93mA$
        - As this value is very close to 1mA, the diode voltage will be very close to the assumed 0.7V

- $= \Rightarrow$ the Q point is $V_D = 0.7V$, $I_D = 0.93mA$
- 

(b)
Figure 4.14
(b) Circuit for calculating the dc operating point.

- ac analysis
  - the Q point is $V_D = 0.7V$, $I_D = 0.93mA$
  - At this operating point, r_d is
    - $r_d = \frac{V_T}{I_D} = \frac{25mV}{0.93mA} = 26.9\Omega$
  - small-signal equivalent circuit can be drawn by replacing 10V voltage source by short-circuit and diode by r_d
    - in the figure
    - v_s is 60Hz 1-V peak sinusoidal component of V^+
    - v_d is the corresponding signal across the diode
    - 

(c)
Figure 4.14
(c) Small-signal equivalent circuit.
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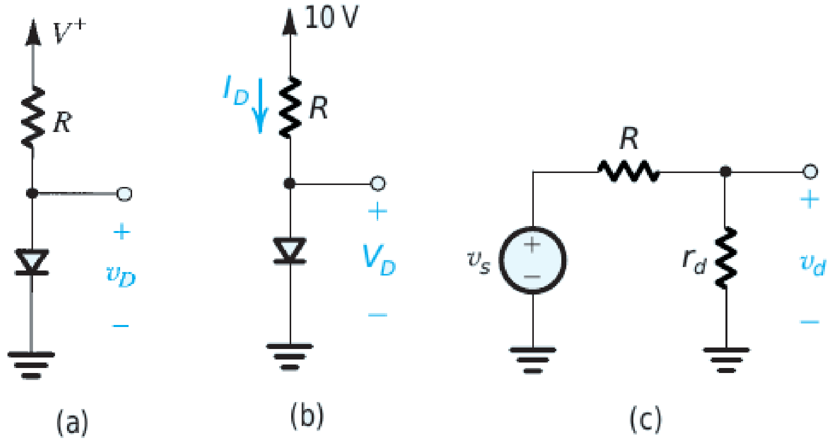


Figure 4.14
(a) Circuit for Example 4.5.
(b) Circuit for calculating the dc operating point.
(c) Small-signal equivalent circuit.

- by voltage divider rule (fig c)
- $v_d = \frac{r_d}{r_d + R} v_s = \frac{26.9}{26.9 + 10k} v_s = \left(2.683m\right) v_s$
- for $v_{s,peak} = 1V$
  - $v_{d,peak} = 2.683mV$
- Note that this $v_{d,peak} < 5mV$
  - $= \Rightarrow$ the small-signal model is valid for this problem

Use of the Diode Forward Drop in Voltage Regulation

A voltage Regulator

- is a circuit which can provide a constant dc voltage between its output terminals
  - inspite of the changes in the load current drawn from the regulator output terminals
    - or change in the dc power supply voltage that feeds the regulator circuit.
  - As the forward-voltage drop of the diode remains
    - almost constant at approximately 0.7V,
    - while the current changes by relatively large amounts,
    - a forward-biased diode can be used as a simple voltage regulator
  - Regulated voltages greater than 0.7V can be obtained by connecting diodes in series.
-

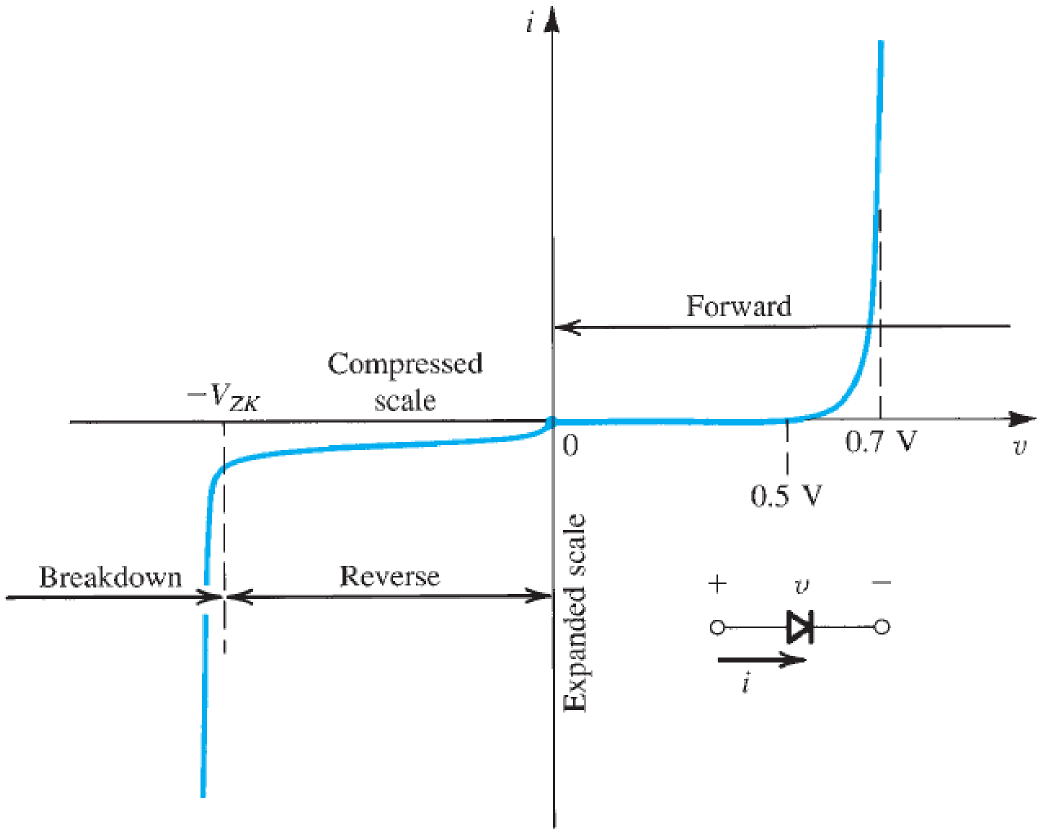


Figure 4.8 The diode i–v relationship with some scales expanded and others compressed in order to reveal details.

Example 4.6

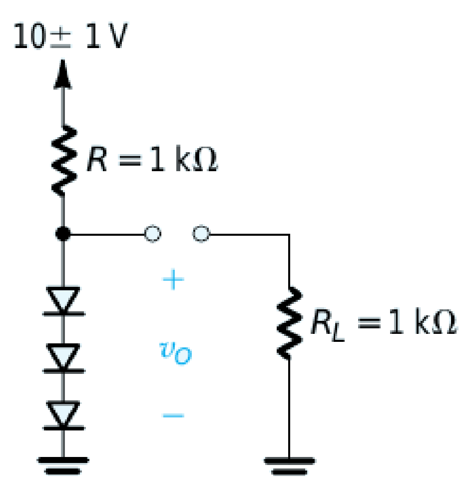
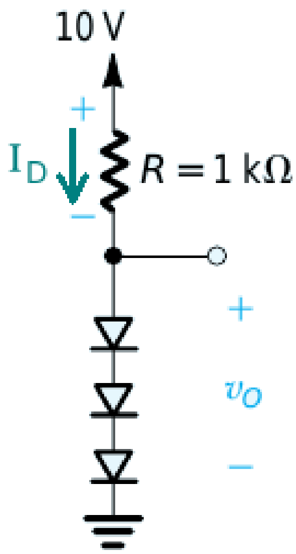
- Consider the circuit shown in Fig. 4.15. A string of three diodes is used to provide a constant voltage of about 2.1 V. We want to calculate the percentage change in this regulated voltage caused by (a) a $\pm 10\%$ change in the power-supply voltage and (b) connection of a 1 $k\Omega$ load resistance.
    - 

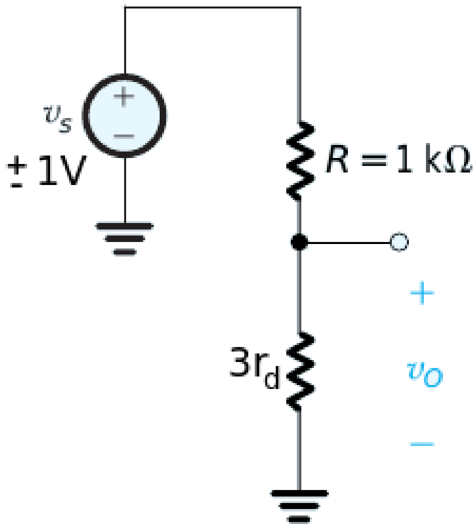
Figure 4.15 Circuit for Example 4.6.

Solution

- no load
    - $V_O = 2.1 V$

    - $I_R = I_D$
- DC analysis
    - ohm's law at R
        - $= \Rightarrow I_D = \frac{10 - V_O}{R} = \frac{10 - 2.1}{1k} = 7.9mA$
        - thus Q point is $I_D = 7.9mA, V_D = \frac{V_O}{3} = \frac{2.1}{3} = 0.7V$
    - 

- Q point is $I_D = 7.9mA, V_D = 0.7V$
- AC analysis
    - $r_d = \frac{V_T}{I_D} = \frac{25m}{7.9m} = 3.2\Omega$
        - ac model is obtained by replacing
        - 10V dc source by short circuit
        - and each diode by a resistor of $3.2\Omega\left(= r_d\right)$
    - using voltage divider
        - $v_o = \frac{3r_d}{3r_d + 1k} v_s = \frac{9.6}{9.6 + 1k} v_s = \left(9.5m\right) v_s$
        - $= \Rightarrow v_{o,peak} = \left(9.5m\right) v_{s,peak}$
        - for $v_{s,peak} = 1V = \Rightarrow v_{o,peak} = 9.5mV$



Example 4.6a

- $v_{s,peak} = 1V = \Rightarrow v_{o,peak} = 9.5mV$
  - the percentage change in supply voltage is = $\frac{1V}{10V} \times 100 = 10\%$
    - the corresponding percentage change in output voltage is = $\frac{9.5mV}{2.1V} \times 100 = 0.45\%$
  - which is quite small
    - $= \Rightarrow$ the diode is providing voltage regulation.
  - i.e. corresponding to $\pm 1V$ ($\pm 10\%$) change in supply voltage,
  - the output voltage changes only by $\pm 9.5mV$ ($\pm 0.45\%$)
  - Note that
    - peak signal voltage across the three diodes is 9.5mV
    - $= \Rightarrow$ the peak signal voltage across each diode is $\frac{9.5mV}{3} = 3.2mV < 5mV$
    - thus the small signal model is valid here

Example 4.6b

- Consider the circuit shown in Fig. 4.15. A string of three diodes is used to provide a constant voltage of about 2.1 V. We want to calculate the percentage change in this regulated voltage caused by (b) connection of a 1 $k\Omega$ load resistance.
  -

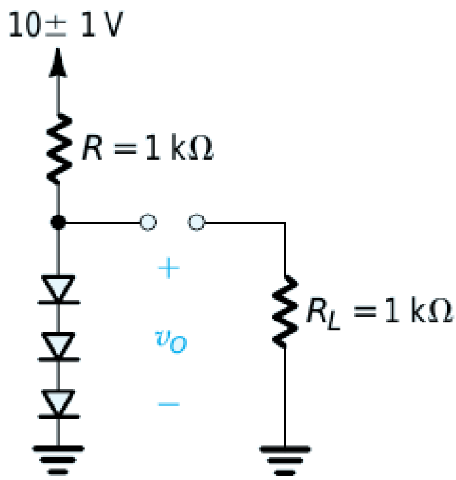


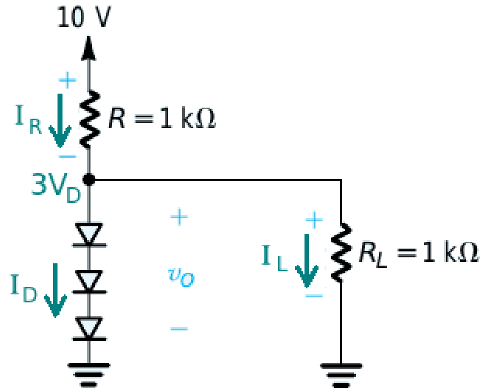
Figure 4.15 Circuit for Example 4.6.

- this part b can be approximately solved by
  - using the small signal model
  - connecting a load resistance of $1k\Omega$
    - $= \Rightarrow$ 1k resistor draws a current of approximately $\frac{2.1}{1k} \approx 2.1mA$
    - $= \Rightarrow$ a decrease in diode current by $2.1mA$ i.e. $\Delta i_d = -2.1mA$
    - $= \Rightarrow$ voltage across each diode changes by

      $\Delta v_d = \Delta i_d r_d = \left(-2.1m\right)3.2 = -6.72mV$
  - $\Delta v_o = 3\Delta v_d = -20.16mV$
  - Note that the voltage change across each diode is 6.72mV>5mV
    - thus using small-signal model in this part b is not entirely justified.

Example 4.6b; continued (Using Exponential Model)

- Accurate solution of this part b can be obtained by
  - using exponential model (graphical analysis or iterative analysis)
  - $I_R = I_D + I_L$
    - $\frac{10-3V_D}{1k} = I_D + \frac{3V_D-0}{1k}$
    - $\frac{10-3V_D}{1k} - \frac{3V_D}{1k} = I_D$
    - $I_D = \frac{10-3V_D-3V_D}{1k} = \frac{10-6V_D}{1k}$
  - and the diode characteristic equation is
    - $V_2 - V_1 = V_T \ln \frac{I_2}{I_1}$

- 

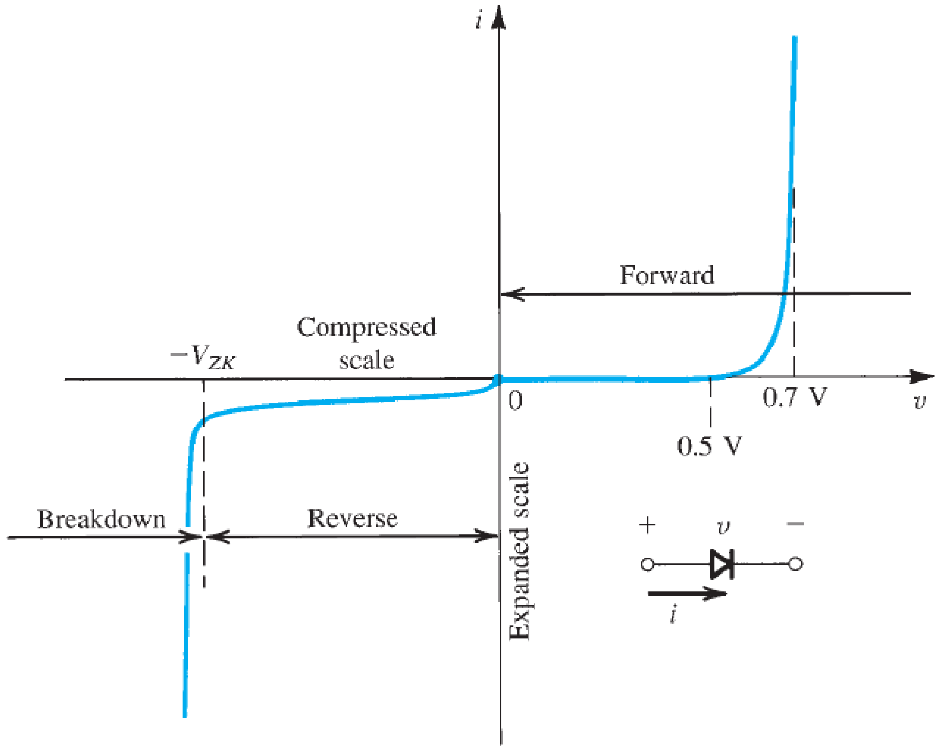
  - for iterative analysis
    - starting point from part (a) is $V_1 = 0.7V$, $I_1 = 7.9mA$
    - let $V_D = 0.7V$

- $I_D = \frac{10 - 6V_D}{1k}$, $V_2 = V_1 + V_T \ln \frac{I_2}{I_1}$
  - for Iteration 1: $V_1 = 0.7V$, $I_1 = 7.9mA$, $V_D = V_1 = 0.7V$

Iteration 1

- $I_D = \frac{10 - 6V_D}{1k} = \frac{10 - 6(0.7)}{1k} = 5.8mA$
  - $V_2 = V_D = V_1 + V_T \ln \frac{I_2}{I_1} = 0.7 + 25m \ln \frac{5.8m}{7.9m} = 0.69227V$
  - for Iteration 2:$V_1 = 0.69227V$, $I_1 = 5.8mA$
  - $I_D = \frac{10 - 6V_D}{1k} = \frac{10 - 6(0.69227)}{1k} = 5.85mA$
  - $V_2 = V_D = V_1 + V_T \ln \frac{I_2}{I_1} = 0.69227 + 25m \ln \frac{5.85m}{5.8m} = 0.69248V$
- Thus after iteration 2, we have $I_D = 5.85mA$, $V_D = 0.69248V$
  - $= \Rightarrow v_O = 3V_D = 2.0774$
  - $\Delta v_o = 2.0774 - 2.1 = -22.6mV$

Operation in the Reverse Breakdown Region - Zener Diodes



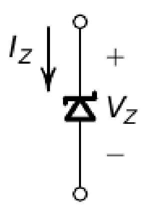
- **Figure 4.8** The diode i–v relationship with some scales expanded and others compressed in order to reveal details.
- In the breakdown region
  - the diode has a very steep i-v curve
  - $= \Rightarrow$ that it can be used for voltage regulation
- A voltage regulator is a circuit
  - that provides a constant dc output voltage
  - inspite of changes in its load current
  - or changes in its feeding power-supply voltage
- For such voltage regulator applications,
  - special diodes are manufactured to operate specifically in the breakdown region
  - such diodes are called zener diodes (or Breakdown diodes)
  - Zener diodes are fabricated with voltages in the range of a few volts to a few hundred volts.
- the circuit symbol for the zener diodes is
  - 

    Figure 4.16 Circuit symbol for a zener diode.
- for Zener diodes
  - current flows into the cathode,
  - and the cathode is +ve with respect to the anode.
  - i.e. the zener diode is reverse biased
- thus I_Z and V_Z both have +ve values

Specifying and Modeling the Zener Diode

- for currents greater than the knee current I_{ZK},
  - the i-v characteristic is almost a straight line.
  - I_{ZK} is specified on the datasheet of the zener diode.
    - Also the voltage across the zener diode V_Z at a
    - specified test current I_{ZT} is given in datasheet.
  - from the fig, one can see that
    - corresponding to current change ΔI,
    - the zener diode voltage changes by ΔV
    -

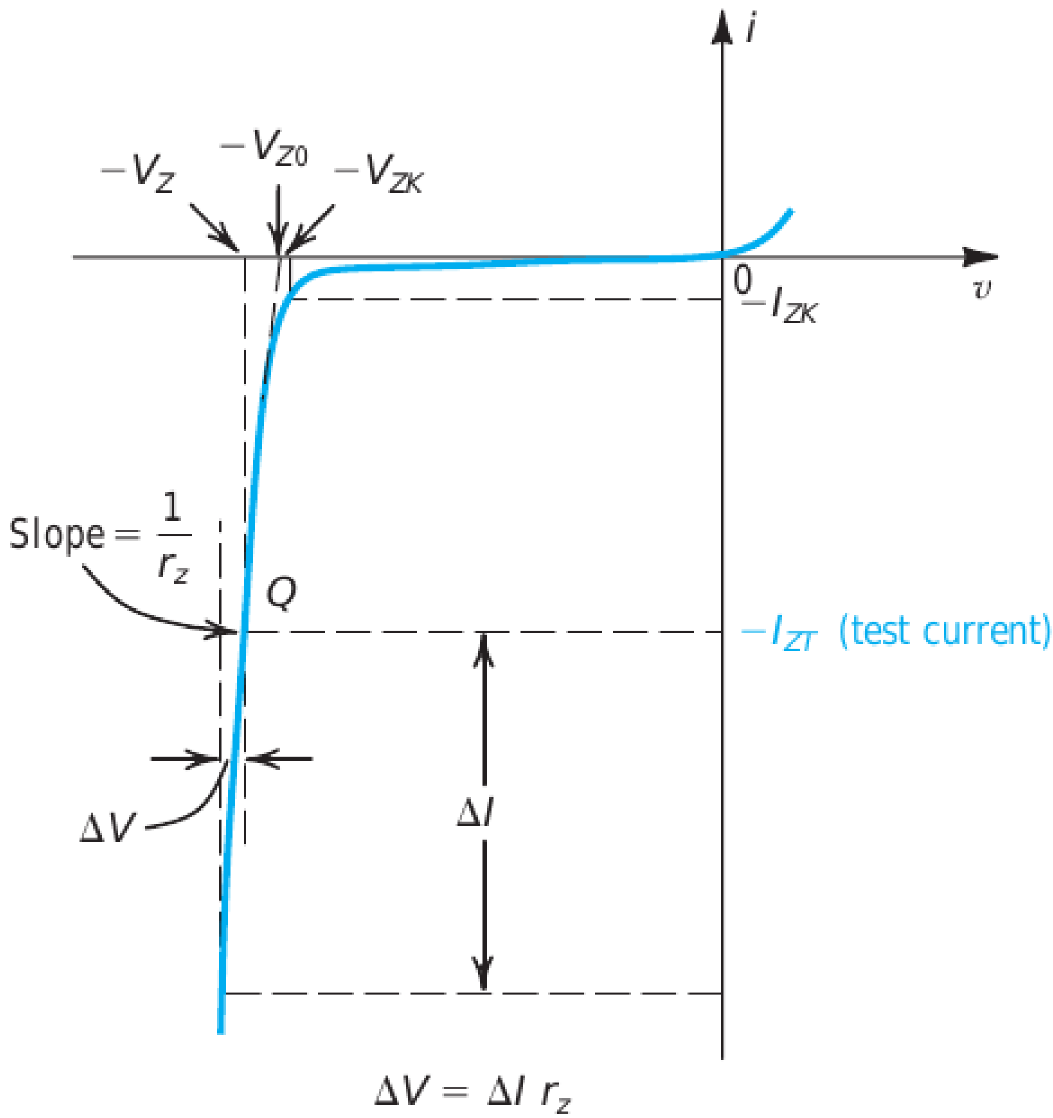


Figure 4.17 The diode i–v characteristic with the breakdown region shown in some detail.

- and this change in voltage can be given as
  - $\Delta V = r_z \Delta I$
- where
  - r_z is the inverse of the slope of the almost linear i-v curve at the operating point Q

- r_z is called the incremental resistance of the zener diode at point Q
- r_z is also called dynamic resistance of the zerner diode at point Q
-

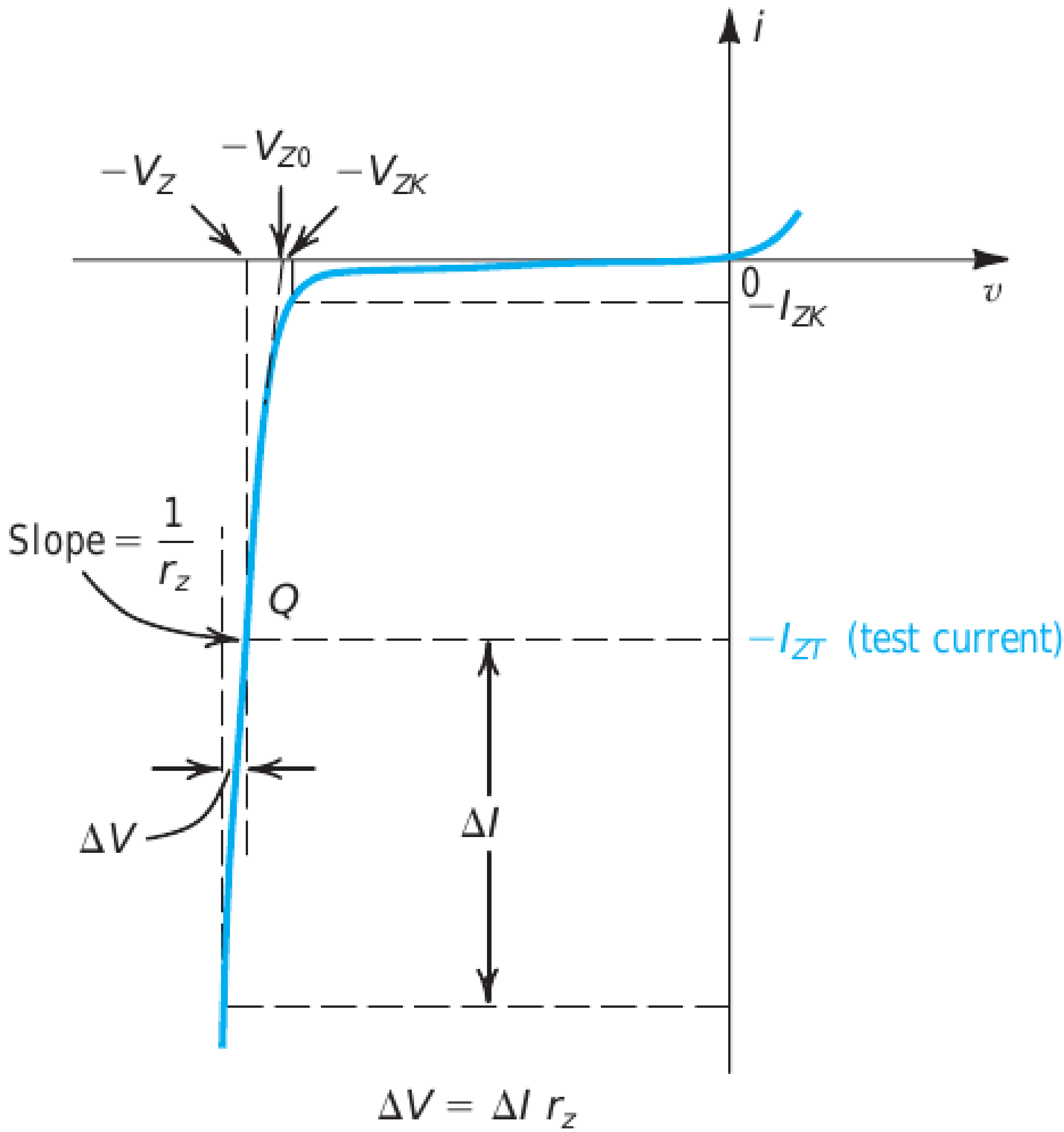


Figure 4.17 The diode i–v characteristic with the breakdown region shown in some detail.

- As a consequence of this almost linear i-v characteristic
  - of the zener diode, it can be modeled as
  -
- here V_{Z0} denotes the point at which the straight line
  - of slope $\frac{1}{r_z}$ intersects the voltage axis

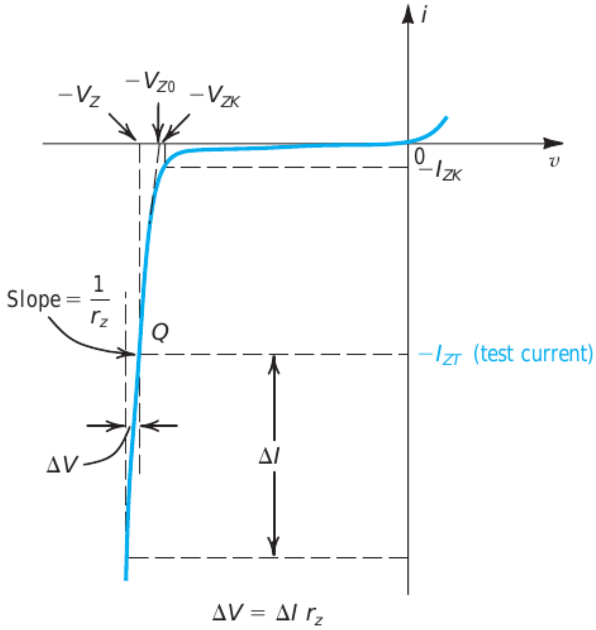
- 

Figure 4.17 The diode *i*–*v* characteristic with the breakdown region shown in some detail.

- V_{Z0} is only slightly different from the knee voltage V_{ZK}
  - in practice, V_{Z0} and V_{ZK} are almost equal.
  -
  - in equation form, this equivalent circuit can be expressed as
- $V_Z = V_{Z0} + r_Z I_Z$
  - where $I_Z > I_{ZK}$ and $V_Z > V_{Z0}$

Use of the Zener as a Shunt Regulator

- zener diodes can be used for voltage regulation
  - when used for voltage regulation
  - a zener diode appears in parallel (or in shunt) with the load
  - thus such a voltage regulator is called a shunt regulator.

Line Regulation

- Line Regulation = $\frac{change\ in\ output\ voltage}{change\ in\ supply\ voltage} = \frac{\Delta V_O}{\Delta V_S}$
  - and is expressed in $\frac{mV}{V}$

Load Regulation

- Load Regulation = $\frac{change\ in\ output\ voltage}{change\ in\ load\ current} = \frac{\Delta V_O}{\Delta I_L}$
  - and is expressed in $\frac{mV}{mA}$

Example 4.7

- The 6.8V zener diode in the circuit of Fig 4.19(a) is specified to have $V_Z = 6.8V$ at $I_Z = 5mA$, $r_Z = 20\Omega$, and $I_{ZK} = 0.2mA$. The supply voltage V^+ is nominally 10V but can vary by $\pm 1V$.

- (a) Find V_O with no load and with V^+ at its nominal value.

Solution

- $V^+ = 10V$
  -
- $V^+ = 10V$
  - zener diode model is
  - $V_{Z0} = ?$
  - As $V_Z = V_{Z0} + r_Z I_Z$
  - $= \Rightarrow\ V_{Z0} = V_Z - r_Z I_Z = 6.8 - 20 \times 5m = 6.7V$
- now we can replace the zener diode with its model
  - $= \Rightarrow$
- (a) Find V_O with no load and with V^+ at its nominal value.

Solution 4.7a

- $V^+ = 10V, I_L = 0$
  - By KVL
- $V^+ = IR + V_{Z0} + Ir_Z$
  - $I\left(R + r_Z\right) = V^+ - V_{Z0}$
  - or $I = I_Z = \frac{V^+ - V_{Z0}}{R + r_Z} = \frac{10 - 6.7}{500 + 20} = 6.35mA$
  - As $V_O = V_Z = V_{Z0} + I_Z r_Z$
  - $V_O = 6.7 + \left(6.35m\right)20 = 6.827V$
  -
- (b) Find the change in V_O resulting from $\pm 1V$ change in V^+.

Solution 4.7b

- Line Regulation $= \frac{\Delta V_O}{\Delta V^+} = ?$
- Line Regulation $= \frac{change\ in\ output\ voltage}{change\ in\ supply\ voltage}$
  - and is expressed in $\frac{mV}{V}$
  - for a $\pm 1V$ change in V^+, the corresponding change in output voltage is
  - $\Delta v_O = \frac{r_Z}{r_Z + R}\Delta V^+ = \frac{20}{20 + 500}\left(\pm 1V\right) = \pm 38.5mV$
  - $= \Rightarrow$ Line Regulation $= \frac{\Delta V_O}{\Delta V^+} = \frac{38.5mV}{1V} = 38.5\frac{mV}{V}$

- (c) Find the change in V_O resulting from connecting a load resistance R_L that draws current $I_L = 1mA$.

Solution 4.7c

- Load Regulation $= \frac{\Delta V_O}{\Delta I_L} = ?$
  - Load Regulation = $\frac{change\ in\ output\ voltage}{change\ in\ load\ current}$
  - and is expressed in $\frac{mV}{mA}$
  - when a load resistance R_L that draws a load current of $I_L = 1mA$ is connected,
- the zener current I_Z will decrease by 1mA
  - the corresponding change in zener voltage is
    - $\Delta V_O = r_Z \Delta I_Z = 20\left(-1mA\right) = -20mV$
    - $= \Rightarrow \frac{\Delta V_O}{\Delta I_L} = \frac{-20mV}{+1mA} = -20\frac{mV}{mA}$
- (d) Find the change in V_O when $R_L = 2k\Omega$.

Solution 4.7d

- KCL $= \Rightarrow I = I_Z + I_L$
  - or $\frac{V^+ - V_O}{R} = \frac{V_O - V_{ZO}}{r_Z} + \frac{V_O}{R_L}$
  - $\frac{V^+}{R} - \frac{V_O}{R} = \frac{V_O}{r_Z} - \frac{V_{ZO}}{r_Z} + \frac{V_O}{R_L}$
- Rearranging
  - $\frac{V^+}{R} + \frac{V_{ZO}}{r_Z} = \frac{V_O}{R} + \frac{V_O}{r_Z} + \frac{V_O}{R_L}$
  - $\frac{V^+}{R} + \frac{V_{ZO}}{r_Z} = V_O\left(\frac{1}{R} + \frac{1}{r_Z} + \frac{1}{R_L}\right)$
  - $V_O = \left(\frac{V^+}{R} + \frac{V_{ZO}}{r_Z}\right) / \left(\frac{1}{R} + \frac{1}{r_Z} + \frac{1}{R_L}\right)$
  - $V_O = \left(\frac{10}{500} + \frac{6.7}{20}\right) / \left(\frac{1}{500} + \frac{1}{20} + \frac{1}{2000}\right)$
  - $V_O = 6.7619V$

- $V_O = 6.7619V$
    - As $V_O > V_{Z0} = 6.7V$, the diode is in breakdown region.
    - $\Delta V_O = 6.7619 - 6.83 = -68.095mV$
-
- (e) Find the value of V_O when $R_L = 0.5k\Omega$

Solution 4.7e

- KCL = $\Rightarrow I = I_Z + I_L$
    - or $\frac{V^+ - V_O}{R} = \frac{V_O - V_{ZO}}{r_Z} + \frac{V_O}{R_L}$
    - $\frac{V^+}{R} - \frac{V_O}{R} = \frac{V_O}{r_Z} - \frac{V_{ZO}}{r_Z} + \frac{V_O}{R_L}$
- Rearranging
    - $\frac{V^+}{R} + \frac{V_{ZO}}{r_Z} = \frac{V_O}{R} + \frac{V_O}{r_Z} + \frac{V_O}{R_L}$
    - $\frac{V^+}{R} + \frac{V_{ZO}}{r_Z} = V_O\left(\frac{1}{R} + \frac{1}{r_Z} + \frac{1}{R_L}\right)$
    - $V_O = \left(\frac{V^+}{R} + \frac{V_{ZO}}{r_Z}\right) / \left(\frac{1}{R} + \frac{1}{r_Z} + \frac{1}{R_L}\right)$
-
- $V_O = \left(\frac{V^+}{R} + \frac{V_{ZO}}{r_Z}\right) / \left(\frac{1}{R} + \frac{1}{r_Z} + \frac{1}{R_L}\right)$
    - here $R = 500\Omega$, $r_Z = 20\Omega$, $V^+ = 10V$, $V_{Z0} = 6.7V$, $R_L = 500\Omega$
    - $V_O = \left(\frac{10}{500} + \frac{6.7}{20}\right) / \left(\frac{1}{500} + \frac{1}{20} + \frac{1}{500}\right)$
    - $V_O = 6.5741V$
- Note that $V_O = V_Z < V_{Z0}$
    - for a zener diode to operate in the breakdown region $V_Z > V_{Z0}$
    - $= \Rightarrow$ the zener diode is cut off (is operating in reverse bias region)
    - $= \Rightarrow V_O = \frac{R_L}{R + R_L} V^+ = \frac{500}{500 + 500} V^+ = \frac{1}{2} V^+$
    - or $V_O = \frac{1}{2} V^+ = 5V$
-
- (f) What is the minimum value of R_L for which the diode still operates in the breakdown region?

Solution 4.7f

- for the zener diode to be in the breakdown region
  - $I_Z > I_{ZK}$, $V_Z > V_{Z0}$
  - thus the lower limit of breakdown region operation is
    - $V_Z = V_{Z0} = 6.7V$
    - and $I_Z = I_{ZK} = 0.2mA$
  - KCL $= \Rightarrow I = I_Z + I_L = 0.2m + I_L$
  - Ohm's Law$= \Rightarrow I = \frac{V^+ - V_O}{500}$
  -
- KCL $= \Rightarrow I = I_Z + I_L = 0.2m + I_L$
  - Ohm's Law$= \Rightarrow I = \frac{V^+ - V_O}{500}$
- As $V^+ = 10 \pm 1V$ and its minimum value is 9V
  - $I = \frac{V^+ - V_O}{500} = 0.2m + I_L$
- $I_L = \frac{V^+ - V_O}{500} - 0.2mA$
  - $I_L = \frac{9 - 6.7}{500} - 0.2mA = 4.4mA$
  - $= \Rightarrow R_L = \frac{V_L}{I_L} = \frac{6.7}{4.4m} = 1.52k\Omega$
-