Relations

Relation

- An employee and his or her salary
- A person and a relative

In mathematics we study relationships such "less than", "is parallel to", "is a subset of, those between a positive integer and one that it divides and so on.

- Relationships between elements of sets are represented using the structure called a relation, which is just a subset of the Cartesian product of the sets.

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Suppose R is a relation from A to B. Then R is a set of ordered pairs where each first element comes from A and each second element comes from B.

That is, for each pair $a \in A$ and $b \in B$, exactly one of the following is true:

1. $(a, b) \in R$ we than say "a is $R-$related to b", written aRb
2. $(a, b) \notin R$ we then say "a is not $R-$related to b".

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Define a relation L from $\mathbf{R}$ to $\mathbf{R}$ as follows: For all real numbers x and y,

$$xLy \Leftrightarrow x < y.$$

Draw the graph of L as a subset of the Cartesian plane $\mathbf{R} \times \mathbf{R}$.

Consider these relations on the set of integers:

$$R_1 = \{(a, b) \mid a \leq b\},$$
$$R_2 = \{(a, b) \mid a > b\},$$
$$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\},$$
$$R_4 = \{(a, b) \mid a = b\},$$
$$R_5 = \{(a, b) \mid a = b + 1\},$$
$$R_6 = \{(a, b) \mid a + b \leq 3\}.$$

Which of these relations contain each of the pairs $(1, 1)$, $(1, 2)$, $(2, 1)$, $(1, -1)$, and $(2, 2)$?

Solution: The pair $(1, 1)$ is in R_1, R_3, R_4, and R_6;

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$(1, -1)$ is in R_2, R_3, and R_6;

and finally, $(2, 2)$ is in R_1, R_3, and R_4.

NUST

Let R be a relation from A to B. Define the inverse relation R^{-1} from B *to* A as follows:

$$R^{-1} = \{(y, x) \in B \times A | (x, y) \in R\}.$$

Let $A = \{2, 3, 4\}$ and $B = \{2, 6, 8\}$ and let R be the "x divides y" relation from A to B: For all $(x, y) \in A \times B$,

$$xRy \Leftrightarrow x|y \qquad x \, divides \, y.$$

State explicitly which ordered pairs are in R and R^{-1}, and draw arrow diagrams for R and R^{-1}

Reflexive Relations

A relation R on a set A is called *reflexive* if $(a, a) \in R$ for every element $a \in A$.

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Which of these relations are reflexive?

Solution: The relations R_3 and R_5 are reflexive because they both contain all pairs of the form (a, a), namely, $(1, 1)$, $(2, 2)$, $(3, 3)$, and $(4, 4)$. The other relations are not reflexive because they do not contain all of these ordered pairs. In particular, R_1, R_2, R_4, and R_6 are not reflexive because $(3, 3)$ is not in any of these relations.

Reflexive Relations

Define a relation R on $\mathbf{R}$ (the set of all real numbers) as follows: For all $x, y \in \mathbf{R}$,

$$xRy \Leftrightarrow x < y.$$

Is R reflexive?

Define a relation T on $\mathbf{Z}$ (the set of all integers) as follows: For all integers m and n,

$$mTn \Leftrightarrow 3 \text{ divides } (m - n)$$

Is T reflexive?

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Symmetric Relations

Let R be a relation on a set A. R is symmetric if, and only if, for all $x, y \in A$, if xRy then yRx.

R is symmetric $\Leftrightarrow$ for all x and y in A, if $(x, y) \in R$ then $(y, x) \in R$.

If any one element is related to any other element, then the second element is related to the first.

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Transitive Relations

Let R be a relation on a set A. R is transitive if, and only if, for all $x, y, z \in A$, if xRy and yRz then xRz.

R is transitive $\Leftrightarrow$ for all x, y and z in A, if $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$.

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Representing Relations Using Matrices

A relation between finite sets can be represented using a zero - one matrix.

Suppose that R is a relation from $A = \{a_1, a_2, \ldots, a_m\}$ to $B = \{b_1, b_2, ..., b_n\}$.The relation R can be represented by the matrix $M_R = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1, & \text{if } (a_i, b_j) \in R; \\ 0, & \text{if } (a_i, b_j) \notin R. \end{cases}$$

Suppose that $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. $(a, b) \in R \Leftrightarrow a > b$.

What is the matrix representing R?

$R = \{(2, 1), (3, 1), (3, 2)\}$

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$$R = \{(2, 1), (3, 1), (3, 2)\} \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

NUST

Representing Relations Using Matrices

A relation between finite sets can be represented using a zero - one matrix.

Suppose that R is a relation from $A = \{a_1, a_2, \ldots, a_m\}$ to $B = \{b_1, b_2, ..., b_n\}$.The relation R can be represented by the matrix $M_R = [m_{ij}]$, where

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Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4, b_5\}$. Which ordered pairs are in the relation R represented by the matrix

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Value of m_{ij} indicates pair (a_i, b_j) belongs to relations or not.

$m_{11} = 0$ implies $(a_1, b_1) \notin R$

$m_{12} = 1$ implies $(a_1, b_2) \in R$

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Matrix for Reflexive Relation

A relation R on A is reflexive if $(a, a) \in R$ whenever $a \in A$.
Thus, R is reflexive if and only if $(a_i, a_i) \in R$ for $i = 1, 2, ..., n$.
Hence, R is reflexive if and only if $m_{ii} = 1$, for $i = 1, 2, ..., n$.
In other words, R is reflexive if all the elements on the main diagonal of M_R are equal to 1,

R on the set $A = \{a_1, a_2, ..., a_n\}$ is symmetric if and only if $(a_j, a_i) \in R$ whenever $(a_i, a_j) inR$.

In terms of the entries of M_R, R is symmetric if and only if $m_{ji} = 1$ whenever $m_{ij} = 1$. This also means $m_{ji} = 0$ whenever $m_{ij} = 0$.

Consequently, R is symmetric if and only if $m_{ij} = m_{ji}$, for all pairs of integers i and j with $i = 1, 2, ..., n$ and $j = 1, 2, ..., n$.

R is symmetric if and only if $M_R = (M_R)^t$[1]

[1] t represents the transpose of a matrix

Matrix for Symmetric Relation

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Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$.
The relations

$$R_1 = \{(1, 1),\ (2, 2),\ (3, 3)\} \text{ and}$$
$$R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$$

can be combined to obtain

$$R_1 \cup R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (3, 3)\},$$
$$R_1 \cap R_2 = \{(1, 1)\},$$
$$R_1 - R_2 = \{(2, 2), (3, 3)\},$$
$$R_2 - R_1 = \{(1, 2), (1, 3), (1, 4)\}.$$

Let R_1 and R_2 be two relations. The matrices representing the union and intersection of these relations are

$$M_{R_1 \bigcup R_2} = M_{R_1} \vee M_{R_2},$$

and

$$M_{R_1 \bigcap R_2} = M_{R_1} \wedge M_{R_2}.$$