Lecture 12

EE-215 Electronic Devices and Circuits

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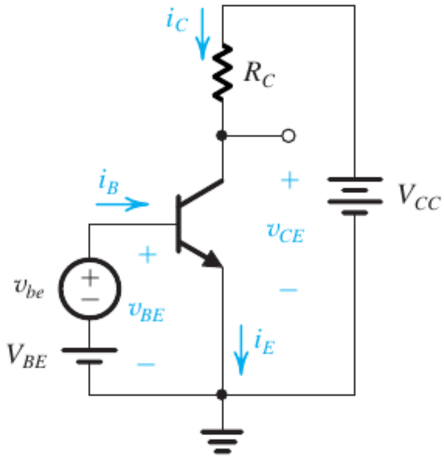
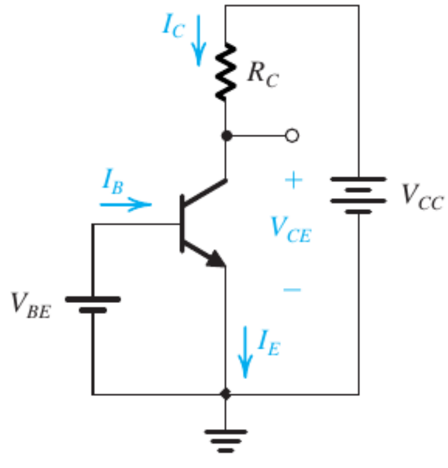
Small-Signal Operation and Models

- Thus for a BJT amplifier, linear amplification can be achieved by
    - biasing the BJT to operate in the active region
    - and by keeping the input signal small
- for the conceptual amplifier circuit shown in fig
    - here the base-emitter junction is forward-biased by a dc voltage V_{BE}
        - the reverse-bias of the collector-base junction is achieved by connecting
            - the collector to another power supply of voltage V_{CC} through a resistor R_C
        - the input signal to be amplified v_{be} is superimposed on the dc bias voltage V_{BE}
        - let's first consider the dc bias conditions by setting the signal $v_{be} = 0$



(a)

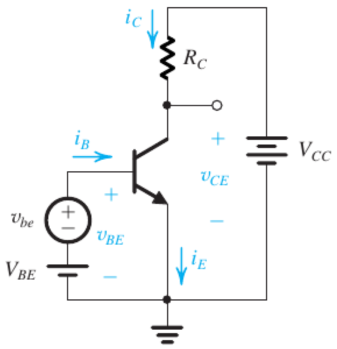
- thus the dc currents and voltages can be given as
    - $I_C = I_S e^{V_{BE}/V_T}$
        - $I_E = I_C/\alpha$
            - $I_B = I_C/\beta$
            - $V_{CE} = V_{CC} - I_C R_C$
        - Note that for active-mode operation, V_C should be greater than $(V_B - 0.4)$ by an amount
            - that allows for the required signal swing at the collector



(b)

The Collector Current and the Transconductance

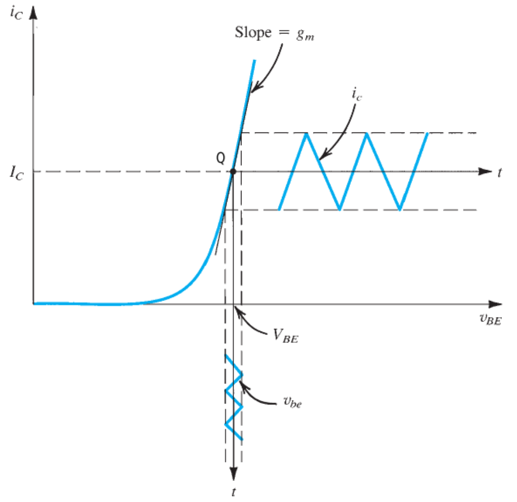
- Now if a signal v_{be} is applied, the total instantaneous base-emitter voltage is
  - $v_{BE} = V_{BE} + v_{be}$
    - and the total instantaneous collector current i_C is
      - $i_C = I_S e^{v_{BE}/V_T} = I_S e^{(V_{BE}+v_{be})/V_T}$
      - $i_C = I_S e^{V_{BE}/V_T} e^{v_{be}/V_T}$
    - As $I_C = I_S e^{V_{BE}/V_T}$
      - $= \Rightarrow i_C = I_C e^{v_{be}/V_T}$



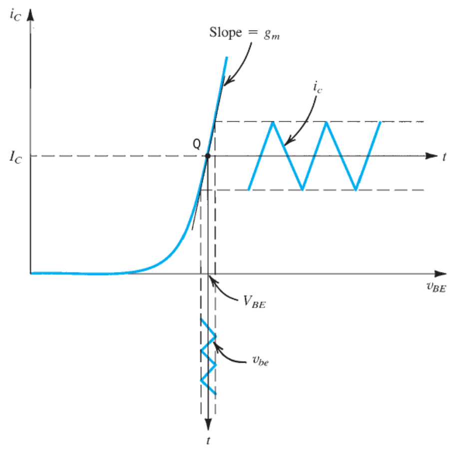
(a)

  - As in terms of series, exponential can be represented as
    - $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$
  - $= \Rightarrow i_C = I_C e^{v_{be}/V_T} = I_C \left(1 + \frac{v_{be}}{V_T} + \frac{1}{2!}\left(\frac{v_{be}}{V_T}\right)^2 + \cdots\right)$
- $i_C = I_C e^{v_{be}/V_T} = I_C \left(1 + \frac{v_{be}}{V_T} + \frac{1}{2!}\left(\frac{v_{be}}{V_T}\right)^2 + \cdots\right)$
  - now if the amplitude of the signal v_{be} is kept sufficiently small i.e. $v_{be} < V_T$ or $\frac{v_{be}}{V_T} < 1$ $= \Rightarrow \left(\frac{v_{be}}{V_T}\right)^2 < \frac{v_{be}}{V_T}$
    - thus we can retain only the 1st two terms, when $v_{be} < V_T$
    - $= \Rightarrow i_C = I_C \left(1 + \frac{v_{be}}{V_T} + \frac{1}{2!}\left(\frac{v_{be}}{V_T}\right)^2 + \cdots\right) \approx I_C \left(1 + \frac{v_{be}}{V_T}\right)$
  - this is the small signal approximation.
  - and under this approximation, the total collector current is
  - $i_C \approx I_C \left(1 + \frac{v_{be}}{V_T}\right) = I_C + \frac{I_C}{V_T} v_{be}$
  - i.e. the collector current is composed of the dc bias value I_C and a signal component i_c,

    - where $i_c = \frac{I_C}{V_T} v_{be}$
- $i_c = \frac{I_C}{V_T} v_{be}$
  - thus the signal current in the collector is proportional to the corresponding base-emitter signal voltage
    - i.e. $i_c = g_m v_{be}$ where $g_m = \frac{I_C}{V_T}$
  - g_m is called the transconductor and is given as $g_m = \frac{I_C}{V_T}$
  - thus the transconductance of the BJT is directly proportional to the collector bias current I_C
  - Note that BJTs have relatively high transconductance as compared to MOSFETs (for a MOSFET, $g_m = \frac{I_D}{V_{OV}/2}$) e.g. at $I_C = 1mA$, $g_m = 40mA/V$
- Graphical interpretation for g_m is as shown in figure
  - Note that g_m is equal to the slope of the $i_C - v_{BE}$ characteristic at the bias point Q
    - i.e. $g_m = . \frac{\partial i_C}{\partial v_{BE}} \Big|_{i_C = I_C}$
    - as $i_C = I_S e^{v_{BE}/V_T}$
      - $g_m = . \frac{\partial i_C}{\partial v_{BE}} \Big|_{i_C = I_C} = . \frac{\partial}{\partial v_{BE}} \left[I_S e^{v_{BE}/V_T} \right] \Big|_{i_C = I_C} = . I_S \frac{\partial}{\partial v_{BE}} \left[e^{v_{BE}/V_T} \right] \Big|_{i_C = I_C}$



  - $g_m = . I_S e^{v_{BE}/V_T} \frac{\partial}{\partial v_{BE}} \left[v_{BE}/V_T \right] \Big|_{i_C = I_C} = . i_C \frac{\partial}{\partial v_{BE}} \left[v_{BE}/V_T \right] \Big|_{i_C = I_C}$
- $g_m = . \frac{i_C}{V_T} \Big|_{i_C = I_C} = \frac{I_C}{V_T}$
  - thus the small-signal approximation implies
    - keeping the signal amplitude sufficiently small that
    - operation is restricted to an almost-linear segment of
    - the $i_C - v_{BE}$ exponential curve.



The Base Current and the Input Resistance at the Base

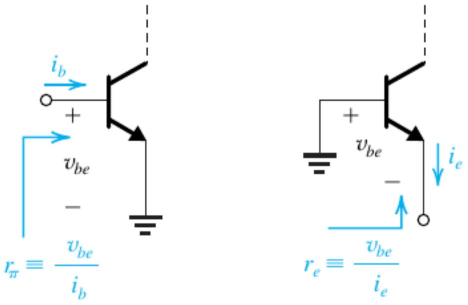
- to determine the resistance seen by v_{be} , first we need to determine the total base current i_B
    - as $i_B = \frac{i_C}{\beta}$
    - using $i_C = I_C + \frac{I_C}{V_T} v_{be}$
    - $= \Rightarrow i_B = \frac{i_C}{\beta} = \frac{I_C + \frac{I_C}{V_T} v_{be}}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta} \frac{I_C}{V_T} v_{be}$
    - or $i_B = I_B + i_b$
        - where $I_B = \frac{I_C}{\beta}$ and i_b is the signal component
        - i.e. $i_b = \frac{1}{\beta} \frac{I_C}{V_T} v_{be} = \frac{g_m}{\beta} v_{be} \because g_m = \frac{I_C}{V_T}$
    - thus $i_b = \frac{g_m}{\beta} v_{be}$
    - The small-signal input resistance between base and emitter, looking into the base is denoted by r_π and is defined as
        - $r_\pi = \frac{v_{be}}{i_b}$
- using $i_b = \frac{g_m}{\beta} v_{be}$
    - $r_\pi = \frac{v_{be}}{i_b} = \frac{v_{be}}{\frac{g_m}{\beta} v_{be}} = \frac{\beta}{g_m}$
    - thus r_π is directly proportional to β and is inversely proportional to g_m
    - As $g_m = \frac{I_C}{V_T}$
        - $= \Rightarrow r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = \frac{V_T}{I_B} \because I_B = \frac{I_C}{\beta}$

The Emitter Current and the Input Resistance at the Emitter

- the total emitter current i_E can be given as
    - $i_E = \frac{i_C}{\alpha} = \frac{I_C + i_c}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha}$
    - $= \Rightarrow i_E = I_E = i_e$
        - where $I_E = \frac{I_C}{\alpha}$
        - and the signal current $i_e = \frac{i_c}{\alpha}$
    - as $i_c = g_m v_{be} = \frac{I_C}{V_T} v_{be}$
        - $= \Rightarrow i_e = \frac{i_c}{\alpha} = \frac{\frac{I_C}{V_T} v_{be}}{\alpha} = \frac{1}{\alpha} \frac{I_C}{V_T} v_{be} = \frac{I_C}{\alpha} \frac{1}{V_T} v_{be} = I_E \frac{1}{V_T} v_{be} \because I_E = I_C / \alpha$
    - thus $i_e = \frac{I_E}{V_T} v_{be}$
    - the small-signal resistance between base and emitter looking into the emitter is denoted by r_e and is defined as
        - $r_e = \frac{v_{be}}{i_e}$
- $r_e = \frac{v_{be}}{i_e}$
    - as $i_e = \frac{I_E}{V_T} v_{be}$
        - $= \Rightarrow r_e = \frac{v_{be}}{i_e} = \frac{v_{be}}{\frac{I_E}{V_T} v_{be}} = \frac{V_T}{I_E}$
    - as $I_E = \frac{I_C}{\alpha}$ and $g_m = \frac{I_C}{V_T}$
        - $= \Rightarrow r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} = \frac{\alpha}{g_m}$
    - thus $r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$

r_π and r_e

- thus $r_\pi = \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = \frac{V_T}{I_B}$ and $r_e = \frac{v_{be}}{i_e} = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$
    - from these relations, we can determine the relation between r_π and r_e

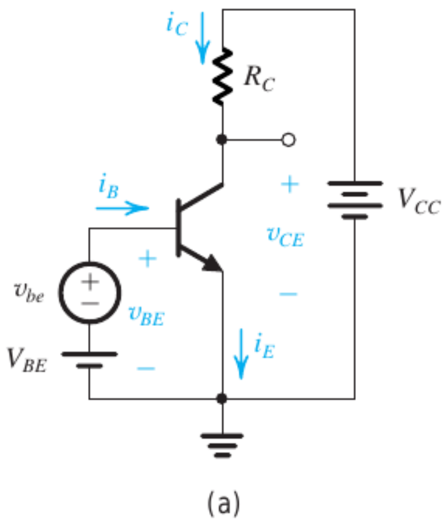


- **Figure 6.38** Illustrating the definition of r_π and r_e.

- i.e. $v_{be} = i_b r_\pi = i_e r_e$
  - $= \Rightarrow r_\pi = \frac{i_e}{i_b} r_e$
- as $\frac{i_e}{i_b} = \frac{i_e}{1}\frac{1}{i_b} = \frac{i_c}{\alpha}\frac{\beta}{i_c} = \frac{i_c(\beta+1)}{\beta}\frac{\beta}{i_c} = \beta + 1$
  - $= \Rightarrow r_\pi = \frac{i_e}{i_b} r_e = \left(\beta + 1\right) r_e$

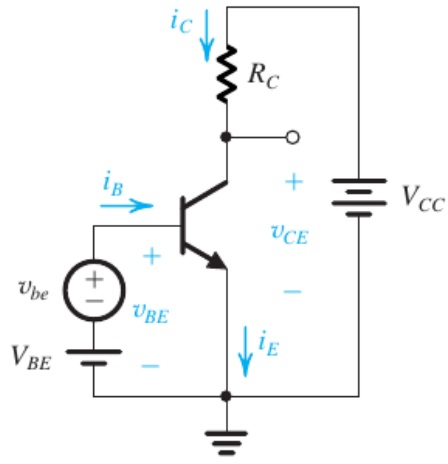
Voltage Gain

- by KVL, $V_{CC} = i_C R_C + v_{CE}$
  - the total instantaneous collector voltage is
    - $v_{CE} = V_{CC} - i_C R_C$
    - under the small signal condition
      - $i_C = I_C + i_c$
    - $= \Rightarrow v_{CE} = V_{CC} - i_C R_C = V_{CC} - (I_C + i_c) R_C$
      - $v_{CE} = V_{CC} - I_C R_C - i_c R_C$
      - $v_{CE} = (V_{CC} - I_C R_C) - i_c R_C$
      - $v_{CE} = V_{CE} - i_c R_C \because V_{CE} = V_{CC} - I_C R_C$
      - here V_{CE} is the dc bias voltage at the collector



(a)

- $v_{CE} = V_{CE} - i_c R_C$
  - $= \Rightarrow$ the signal component of v_{CE} is
    - $v_{ce} = -i_c R_C$
    - As $i_c = g_m v_{be}$
      - $= \Rightarrow v_{ce} = -i_c R_C = -g_m v_{be} R_C$
    - or $v_{ce} = (-g_m R_C) v_{be}$
    - the voltage gain of this amplifier A_v can be given as
    - $A_v = \frac{v_{ce}}{v_{be}} = -g_m R_C$



(a)

- Note that the gain is proportional to g_m . As $g_m = \frac{I_C}{V_T}$, thus stable gain can be achieved by making the collector bias current stable

- $A_v = \frac{v_{ce}}{v_{be}} = -g_m R_C$
  - As $g_m = \frac{I_C}{V_T}$
    - $= \Rightarrow A_v = \frac{v_{ce}}{v_{be}} = -g_m R_C = -\frac{I_C R_C}{V_T}$
  - here the minus sign indicates that the output signal v_{ce} is 180^o out of phase w.r.t. the input signal v_{be}

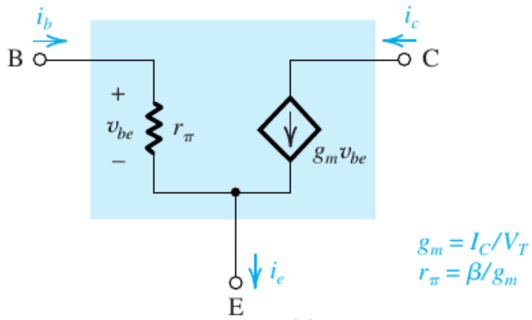
Separating the Signal and the DC Quantities

- thus under the small-signal approximation,
  - signal quatities are superimposed on dc quantities
  - i.e. every current and voltage in the amplifier circuit is composed of
    - two components: a dc component and a signal component
    - thus $v_{BE} = V_{BE} + v_{be}$, $i_C = I_C + i_c$, $i_E = I_E + i_e$,
    - $i_B = I_B + i_b$ and $v_{CE} = V_{CE} + v_{ce}$
  - thus we can simplify analysis by separating “dc calculations” from “small-signal or ac calculations”
  - first we perform dc analysis by suppressing all ac sources, and determine the dc operating point
  - for ac analysis, we suppress all dc sources,
    - i.e. dc voltage source is replaced by a short circuit
    - a dc current source is replaced by an open circuit

The Hybrid-π Model

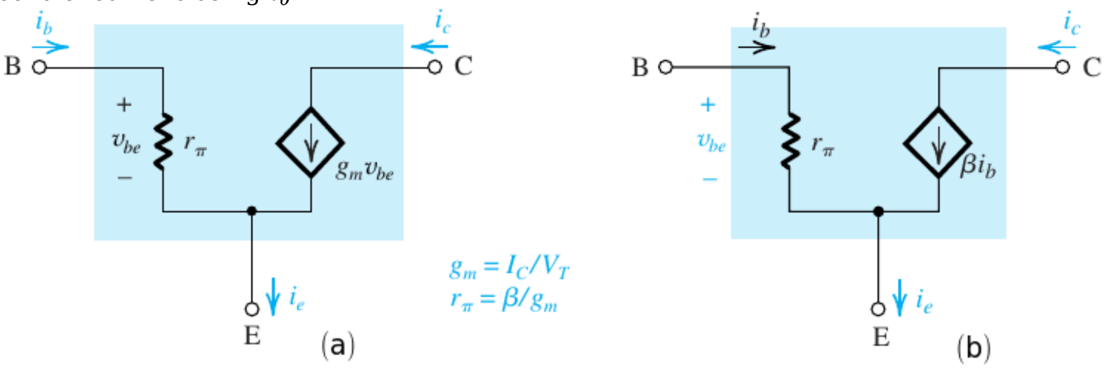
- we have already seen that under the small-signal condition
  - $i_c = g_m v_{be}$ where $g_m = I_C/V_T$
  - $= \Rightarrow$ BJT behaves as a voltage controlled current source
    - it accepts a signal v_{be} between base and the emitter
    - and provides a current $g_m v_{be}$ at the collector terminal
  - The small-signal input resistance between the base and emitter, looking into the base is r_π
  - the output resistance is infinite (if base-width modulation effect is neglected)
- thus the small-signal operation of the BJT can be represented by figure
  - from this model, $i_c = g_m v_{be}$, $i_b = v_{be}/r_\pi$
  - by KCL
    - $i_e = i_b + i_c = \frac{v_{be}}{r_\pi} + g_m v_{be}$
      - $i_e = \left(\frac{1}{r_\pi} + g_m\right) v_{be} = \left(1 + g_m r_\pi\right) \frac{v_{be}}{r_\pi}$

        - As $r_\pi = \frac{\beta}{g_m} \implies g_m r_\pi = \beta$
            - $i_e = \left(1 + g_m r_\pi\right) \frac{v_{be}}{r_\pi} = \left(1 + \beta\right) \frac{v_{be}}{r_\pi}$
        - but $r_\pi = (\beta + 1) r_e$ or $\frac{r_\pi}{\beta+1} = r_e$

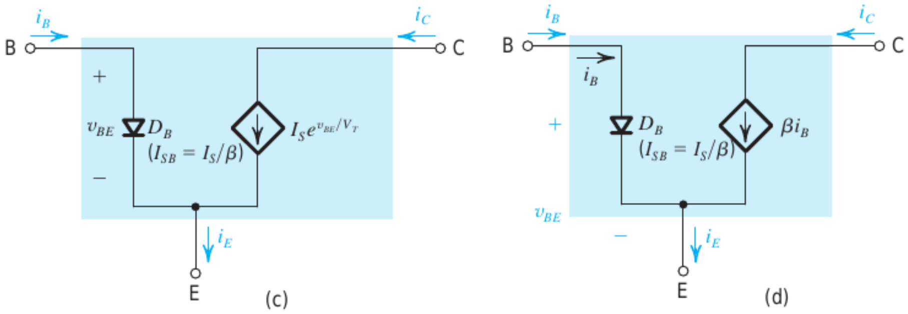


(a)

    - $\implies i_e = \left(1 + \beta\right) \frac{v_{be}}{r_\pi} = \frac{v_{be}}{r_\pi/(1+\beta)} = \frac{v_{be}}{r_e}$
- A slightly different equivalent-circuit model is obtained by
    - expressing the current of the controlled source $(g_m v_{be})$ in terms of the base current i_b
    - i.e. $i_c = g_m v_{be} = g_m (i_b r_\pi) = (g_m r_\pi) i_b = \beta i_b \because r_\pi = \beta/g_m$
    - this results in an alternative equivalent-circuit model shown in fig (b)
    - here the transistor is represented by a current-controlled current source, with the control current being i_b



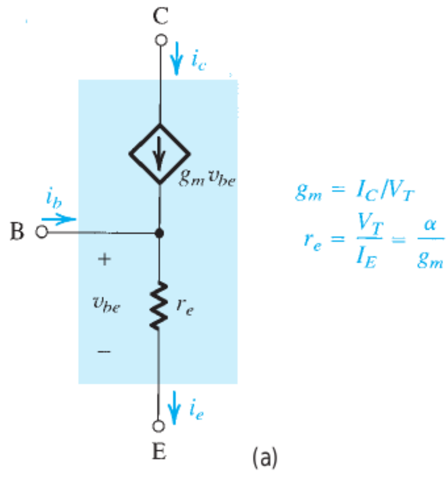
- Note that these models (fig above) are small-signal versions of large signal models (fig below)
    - Note that r_π is the incremental resistance of D_B



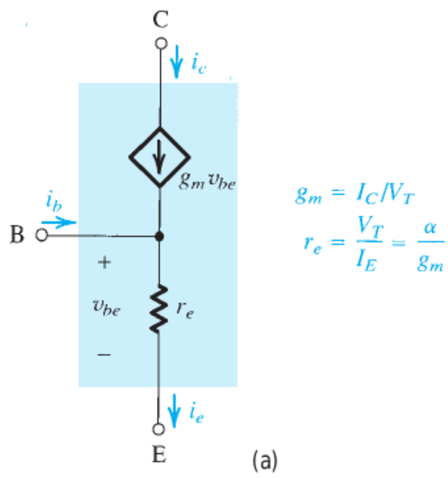
The T Model

- Another possible small-signal model for a BJT called the T-model is shown in figure
    - here the BJT is represented as a voltage-controlled current source
        - with the control voltage being v_{be}
        - here the resistance between base and emitter, looking into the emitter is explicitly shown
        - from the figure we can see that the model yields correct expressions for i_c , i_b and i_e
        - i.e. $i_c = g_m v_{be}$, $i_e = v_{be}/r_e$

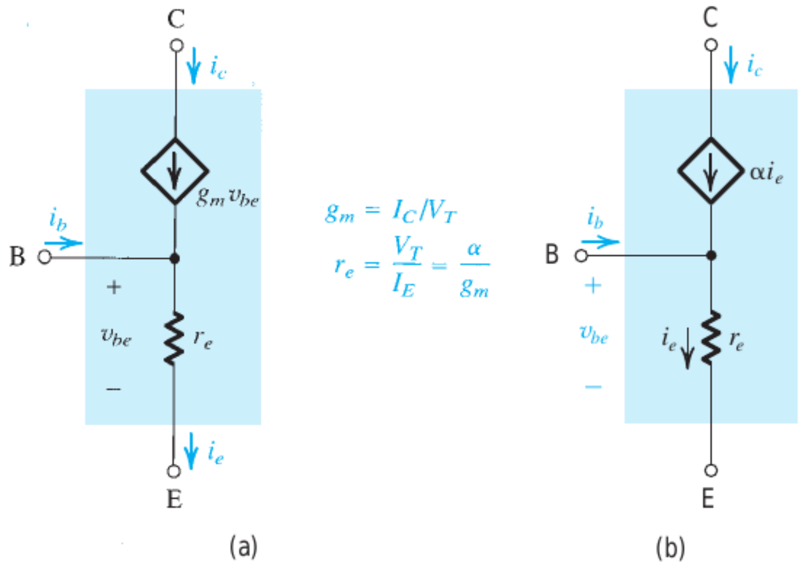
- and $i_b = i_e - i_c = \frac{v_{be}}{r_e} - g_m v_{be} = \left(\frac{1}{r_e} - g_m\right) v_{be}$



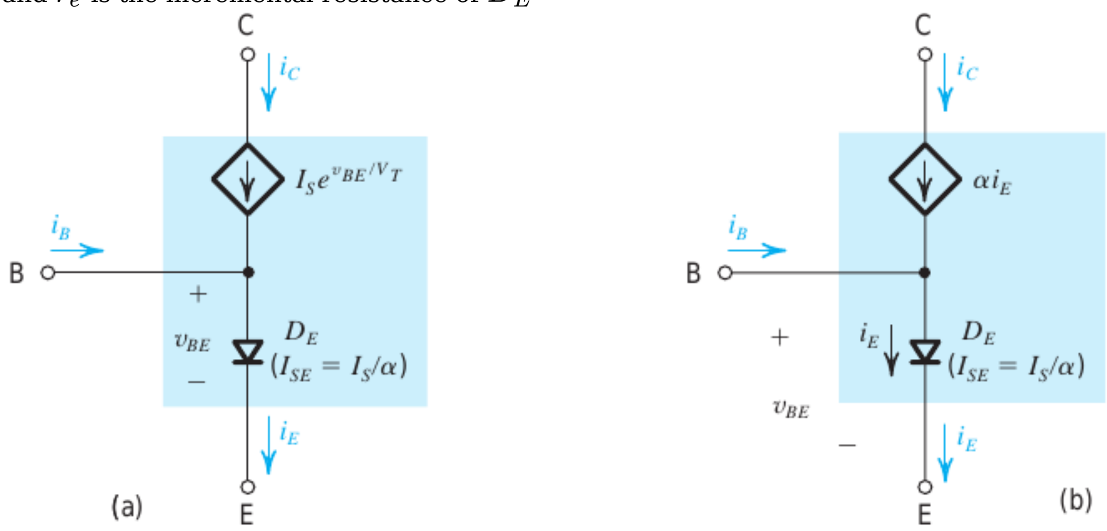
- $i_b = \left(\frac{1}{r_e} - g_m\right) v_{be}$
  - $i_b = \left(\frac{1-g_m r_e}{r_e}\right) v_{be} = \left(\frac{1-\alpha}{r_e}\right) v_{be} \because r_e = \frac{\alpha}{g_m}$ or $g_m r_e = \alpha$
    - $i_b = \left(1 - \alpha\right) \frac{1}{r_e} v_{be} = \left(1 - \frac{\beta}{\beta+1}\right) \frac{1}{r_e} v_{be}$
    - $i_b = \left(\frac{(\beta+1)-\beta}{\beta+1}\right) \frac{1}{r_e} v_{be} = \left(\frac{1}{\beta+1}\right) \frac{1}{r_e} v_{be}$
    - As $r_\pi = (\beta + 1) r_e$
      - $= \Rightarrow i_b = \frac{1}{r_\pi} v_{be}$



- A slightly different equivalent-circuit model is obtained by
  - expressing the current of the controlled source $(g_m v_{be})$ in terms of the emitter current i_e
  - i.e. $i_c = g_m v_{be} = g_m(i_e r_e) = (g_m r_e) i_e = \alpha i_e \because r_e = \alpha / g_m$
  - this results in an alternative equivalent-circuit model (T-model) shown in fig (b)
  - here the BJT is represented by a current-controlled current source, with the control current being i_e

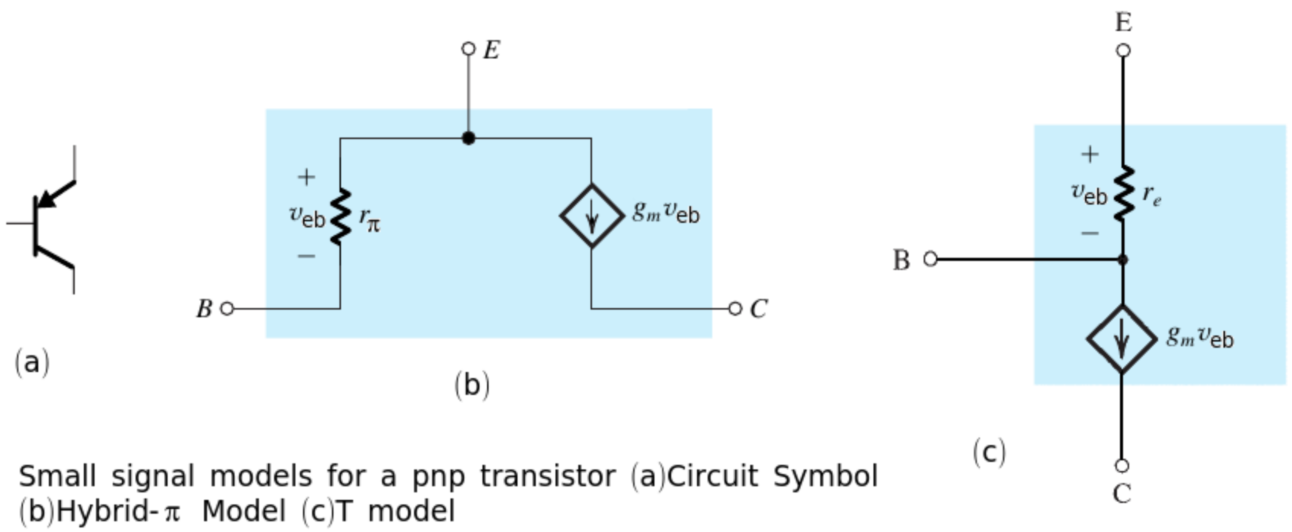


- Note that these small-signal models are small-signal versions of the large-signal models (fig below)
    - and r_e is the incremental resistance of D_E



Small-Signal Models of the pnp Transistor

- for a pnp transistor,



Small signal models for a pnp transistor (a)Circuit Symbol (b)Hybrid-π Model (c)T model
The base-width modulation effect can be included by placing r_o between C and E

- Recall that for a pnp transistor, larger signal models can be given as

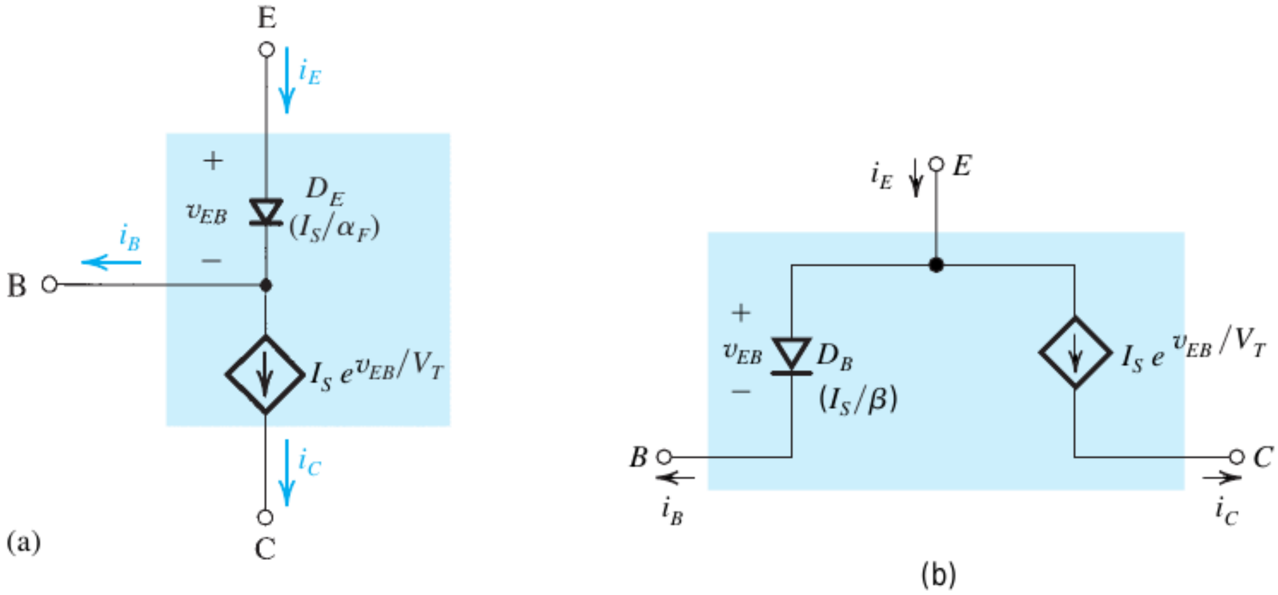
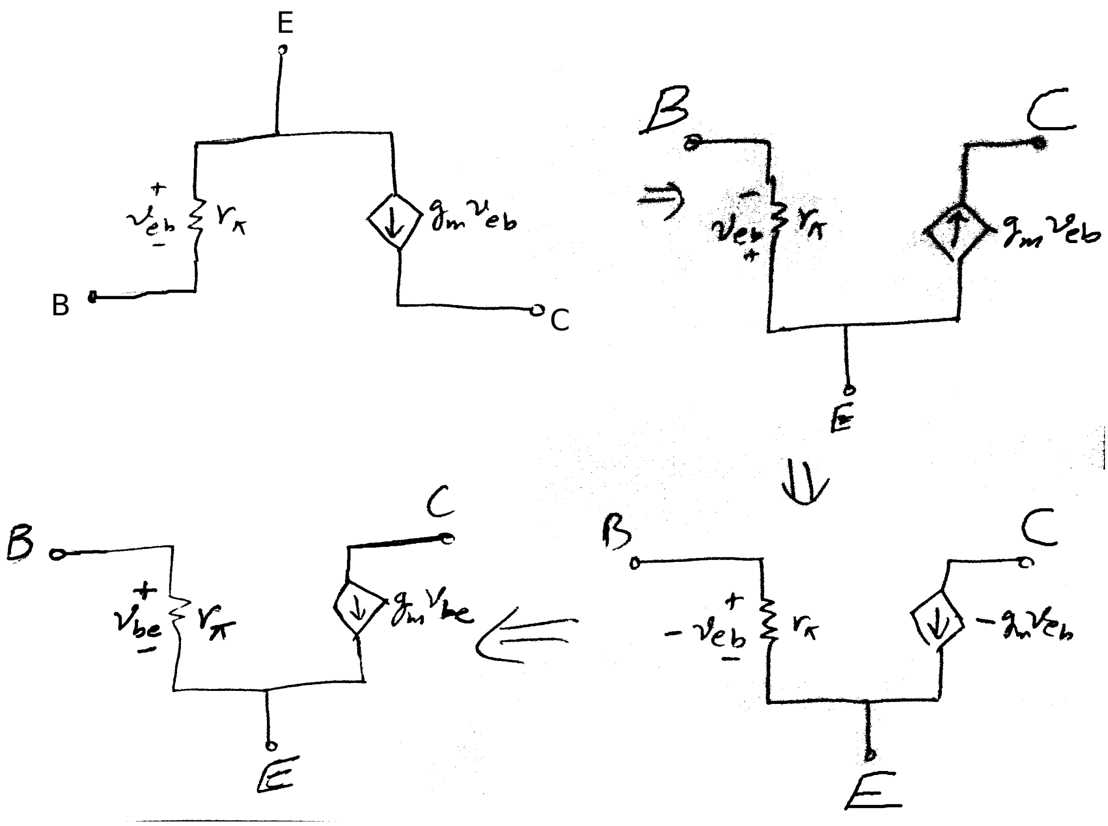


Figure 6.11 Two large-signal models for the *pnp* transistor operating in the active mode.

- Also the above small-signal models developed for npn transistors,
    - apply equally well to the pnp transistor with no change in polarities.
    - to see why?



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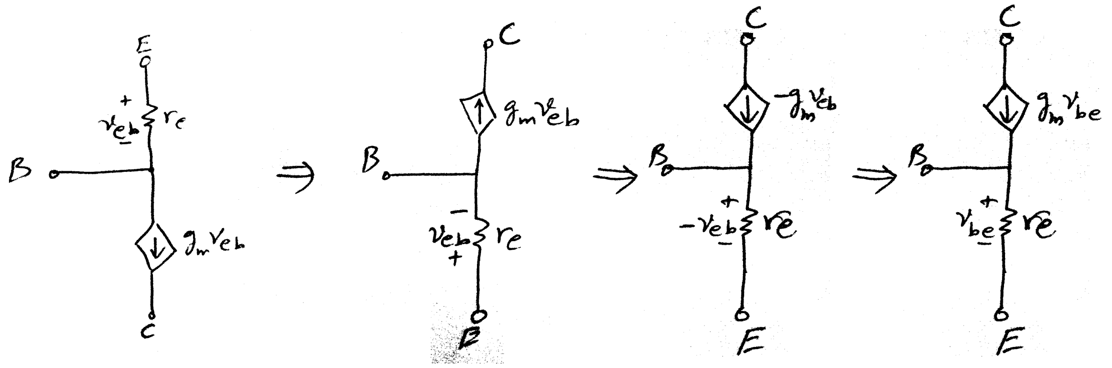
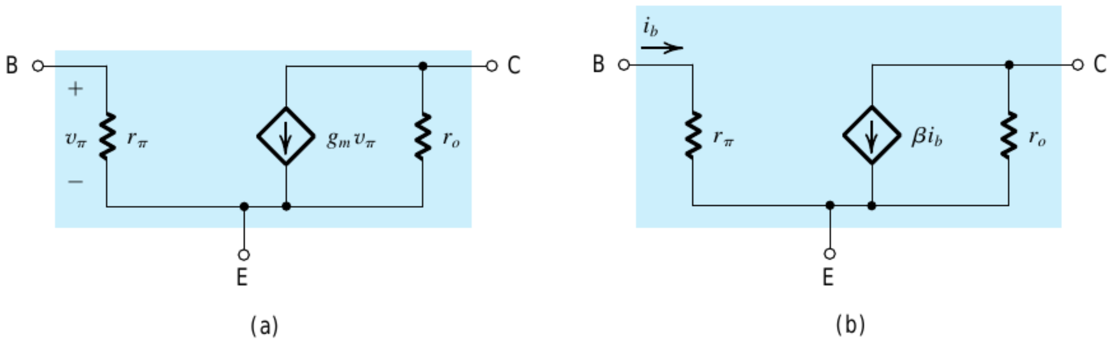
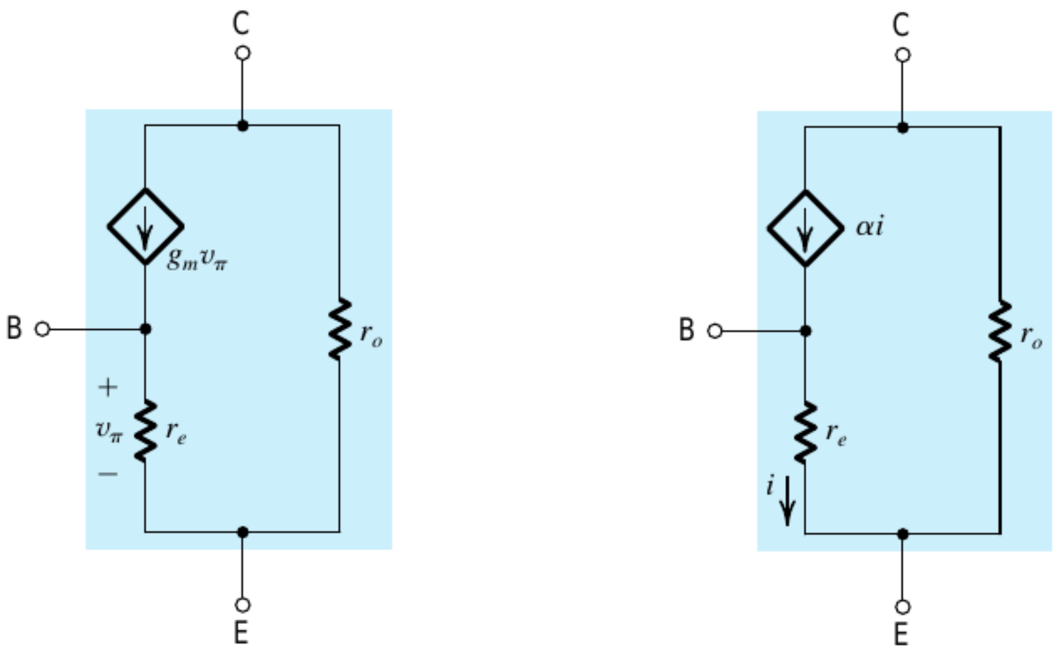
- 

Augmenting the Small-signal Models to account for the Early Effect

- because of Early effect, i_C depends not only on v_{BE} and also on v_{CE}
  - i.e. $i_C = I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$
  - As a consequence of Early effect, the output resistance looking into the collector is not infinite and can be given as
  - $r_o = \frac{V_A}{I_C\text{'}}$
  - where V_A is the Early voltage and I_C' is the collector current with the Early effect neglected.
  - The Early effect can be included in the small-signal models by placing r_o between the collector and the emitter terminals



  - **Figure 6.47** The hybrid-π small-signal model, in its two versions, with the resistance r_o included.
  - 

Application of the Small-Signal Equivalent Circuits

- Thus in the analysis of a BJT amplifier circuit, we follow the following steps
  1. Suppress the signal sources and determine the dc operating point of the BJT (particularly the dc collector current, I_C)
  2. Calculate the values of the small signal model parameters:

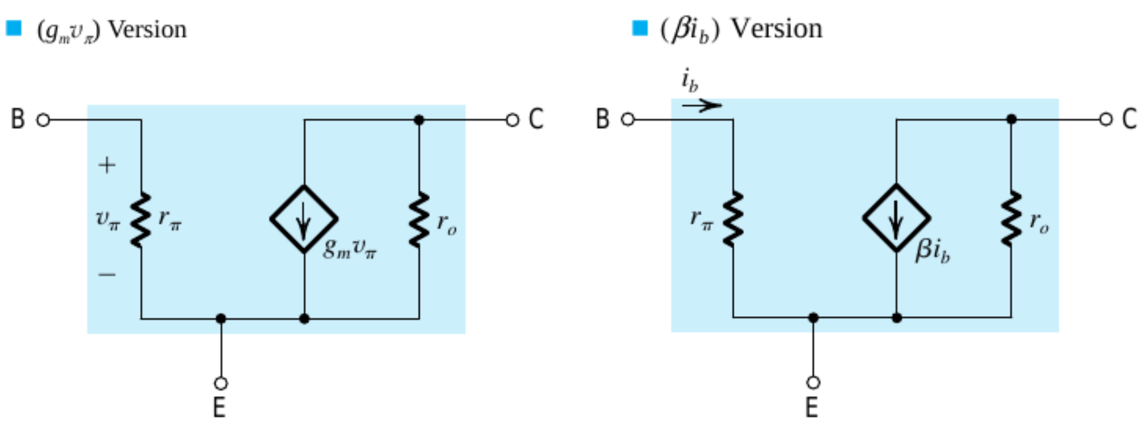
  - i.e. $g_m = \frac{I_C}{V_T}$, $r_\pi = \frac{\beta}{g_m}$, $r_e = \frac{\alpha}{g_m}$

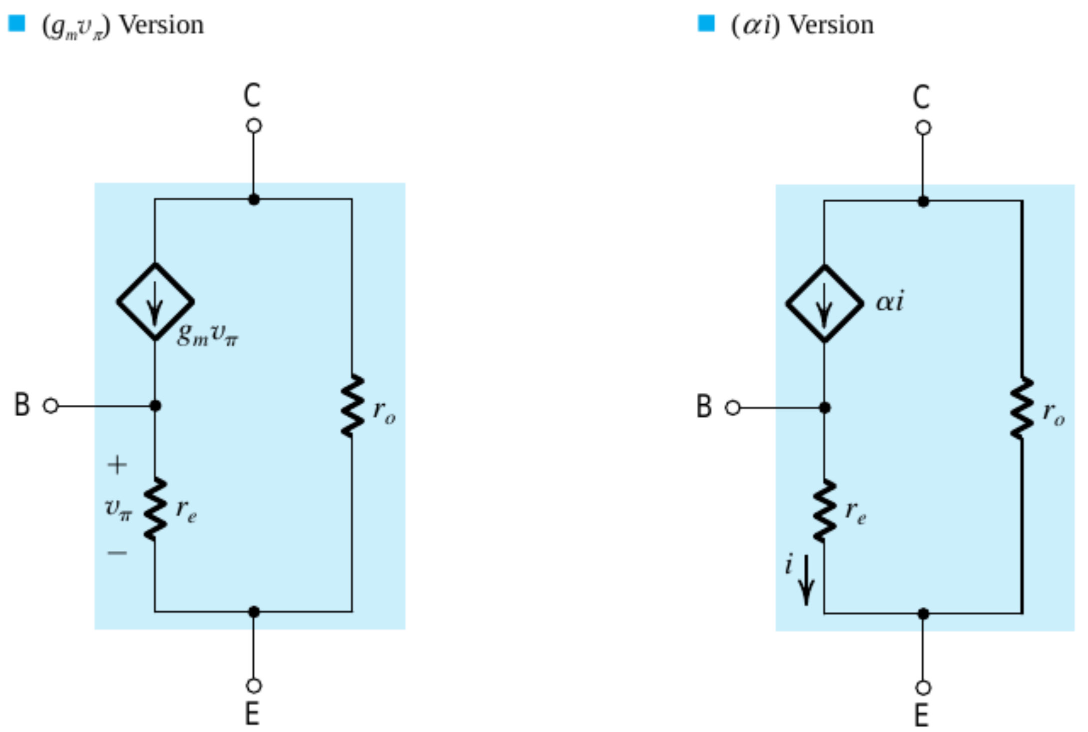
3. Eliminate the dc sources by replacing each dc voltage source with a short-circuit and each current source with an open-circuit
4. Replace the BJT by its small-signal equivalent circuit model
5. Analyse the resulting circuit to determine the required signal quantities e.g. A_v , R_{in} etc.

- Table summarizes the small-signal models and relevant relations for a BJT
  - note that these small-signal models and relations apply equally well to both the npn and the pnp transistors with no change in polarities

Table 6.4 Small-Signal Models of the BJT

Hybrid-π Model





Model Parameters in Terms of DC Bias Currents

$$g_m = \frac{I_C}{V_T} \qquad r_e = \frac{V_T}{I_E} = \alpha\frac{V_T}{I_C} \qquad r_\pi = \frac{V_T}{I_B} = \beta\frac{V_T}{I_C} \qquad r_o = \frac{|V_A|}{I_C}$$

In Terms of g_m

$$r_e = \frac{\alpha}{g_m} \qquad r_\pi = \frac{\beta}{g_m}$$

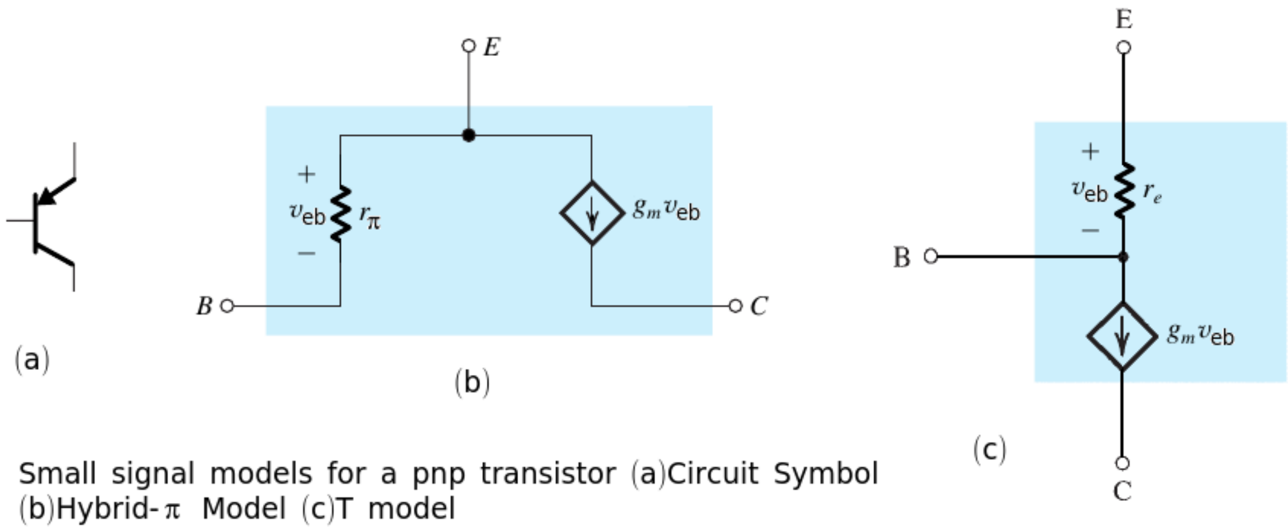
In Terms of r_e

$$g_m = \frac{\alpha}{r_e} \qquad r_\pi = (\beta+1)r_e \qquad g_m + \frac{1}{r_\pi} = \frac{1}{r_e}$$

Relationships between α and β

$$\beta = \frac{\alpha}{1-\alpha} \qquad \alpha = \frac{\beta}{\beta+1} \qquad \beta+1 = \frac{1}{1-\alpha}$$

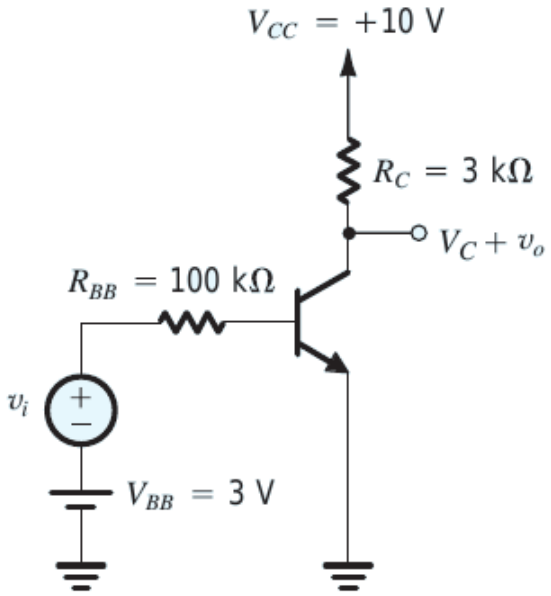
- for a pnp transistor,



Small signal models for a pnp transistor (a)Circuit Symbol (b)Hybrid-π Model (c)T model
The base-width modulation effect can be included by placing r_o between C and E

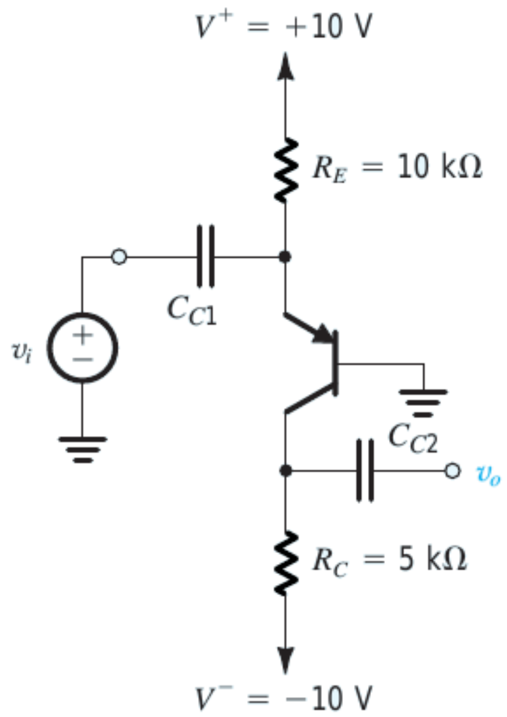
Example 6.14

- Analyze the transistor amplifier shown in Fig 6.42(a) to determine its voltage gain v_o/v_i. Assume $\beta = 100$.



Example 6.16

- Analyze the transistor amplifier shown in Fig 6.44(a) to determine its voltage gain v_o/v_i. Assume $\beta = 100$.



(a)

◦
