Lecture 3b

EE-215 Electronic Devices and Circuits

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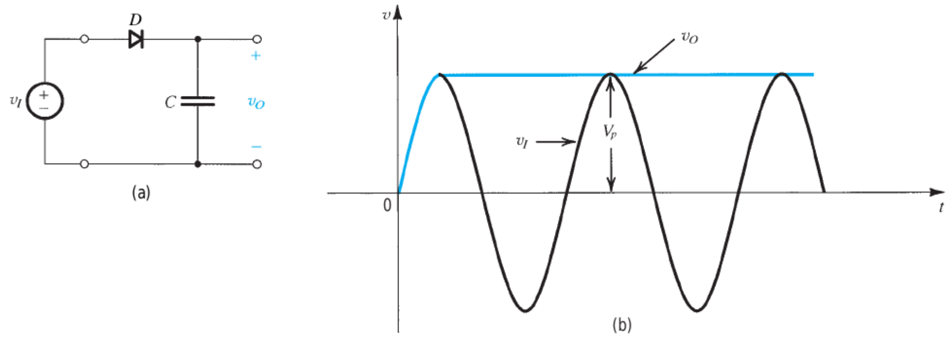
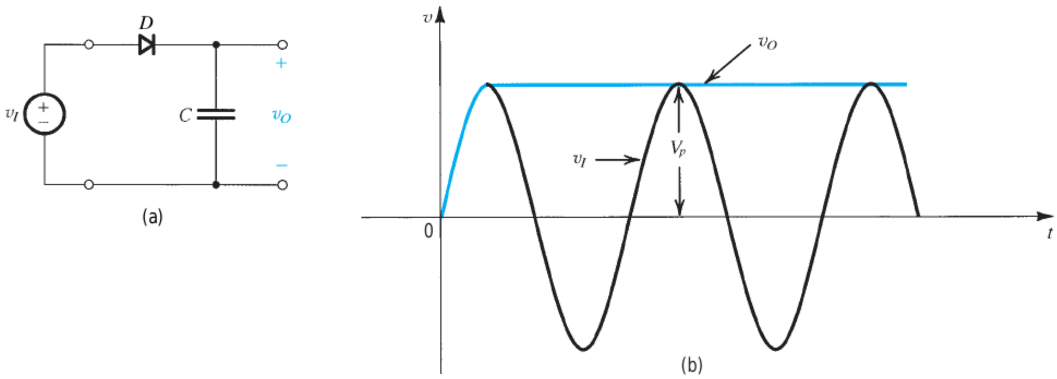
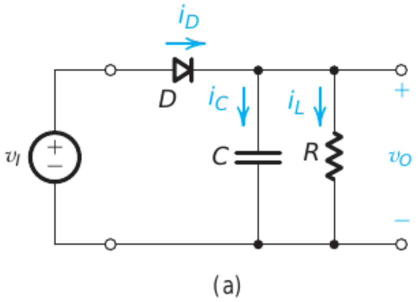
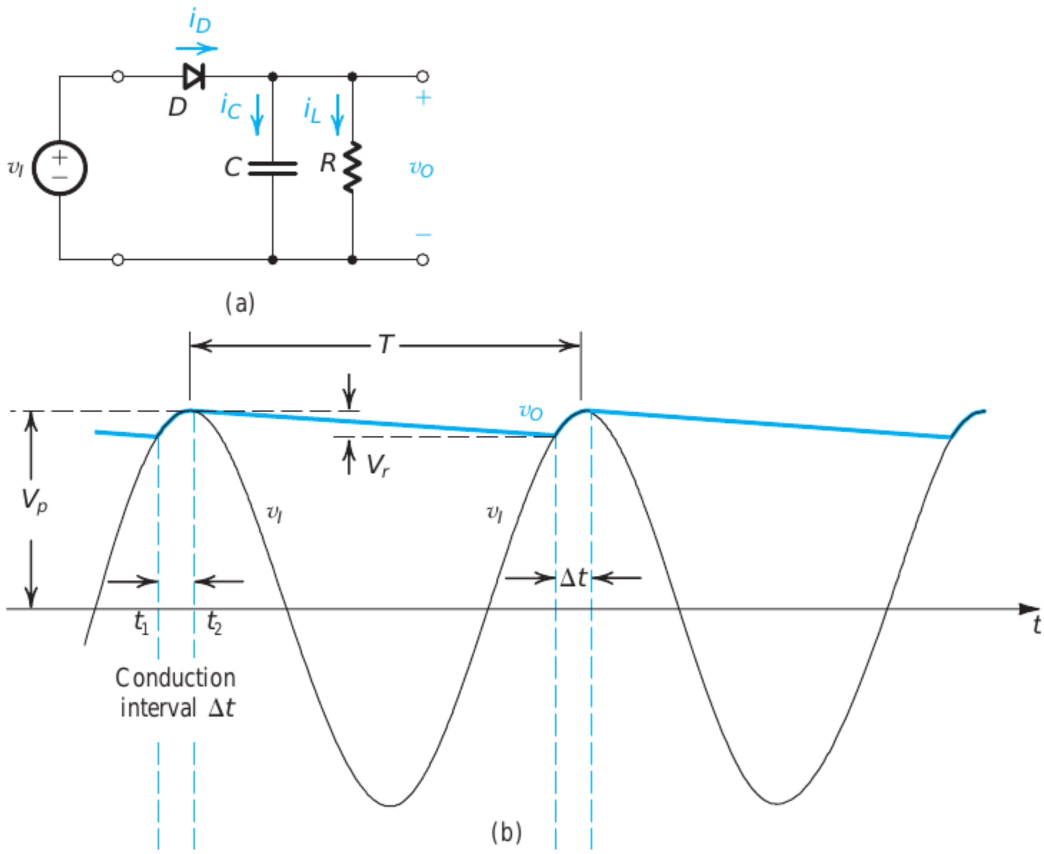
The Rectifier with a Filter Capacitor - The Peak Rectifier

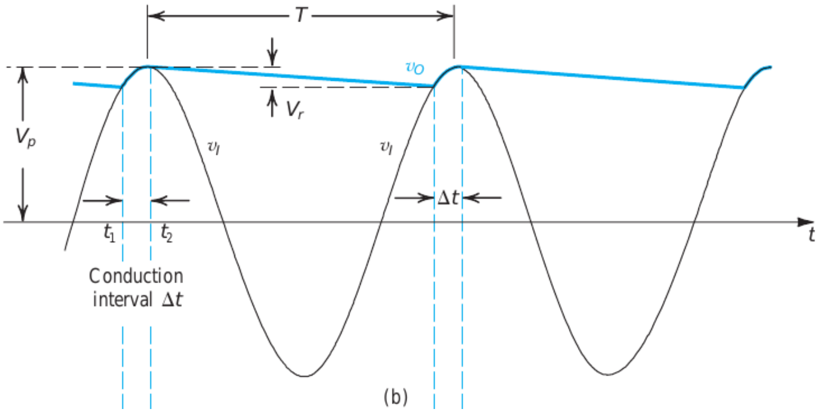
- The rectifier circuits produce a unipolar output voltage,
    - which has a non-zero average component.
    - but because of the pulsating nature of this output,
        - it cannot be readily used as a dc source for electronic circuits
    - to reduce the variations of the output voltage,
        - a capacitor can be placed across the load
    - to see how a capacitor can reduce the variations,
        - first consider the simpler circuit without the load resistor
- here v_I is a sinusoid with a peak value of V_P
    - for simplicity, let's assume that the diode is ideal
    - 

    - as v_I goes positive, the diode becomes forward-biased
        - the diode conducts current and charge the capacitor so that $v_O = v_I$
        - this will continue until v_I reaches its peak value of V_P
- 

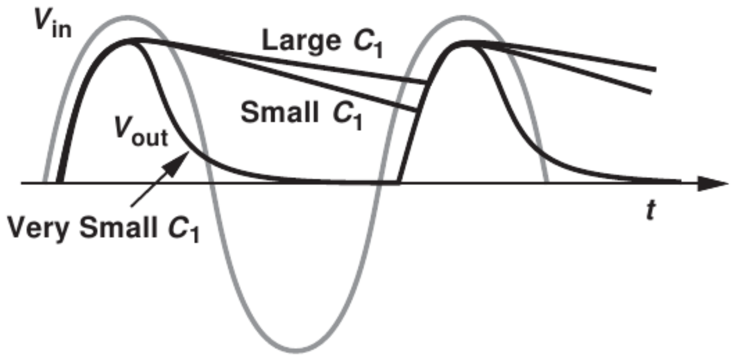
    - beyond V_P, v_I decreases the diode becomes reverse-biased
        - and the output voltage remains constant at V_P

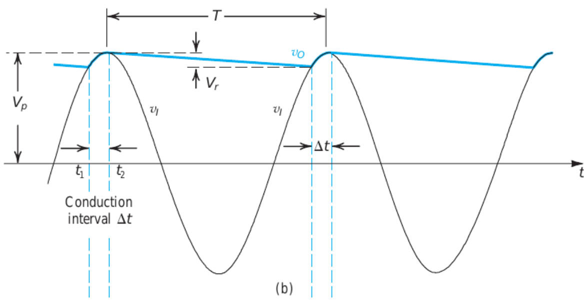
- thus this circuit provides a dc voltage output
    - equal to the peak of the input sinusoid
- Now, consider the more practical situation
  - 

  - here a load resistor R is connected across the capacitor C
  - for simplicity, we will use the ideal diode model.
  - now, for a sinusoidal input v_I, as v_I increases,
    - the diode conduct and charge capacitor until $v_I = V_P$
    - thus the capacitor is charges to the peak of V_P
  - then the diode cuts off,
    - and the capacitor discharges through the load resistor R
- 

- 

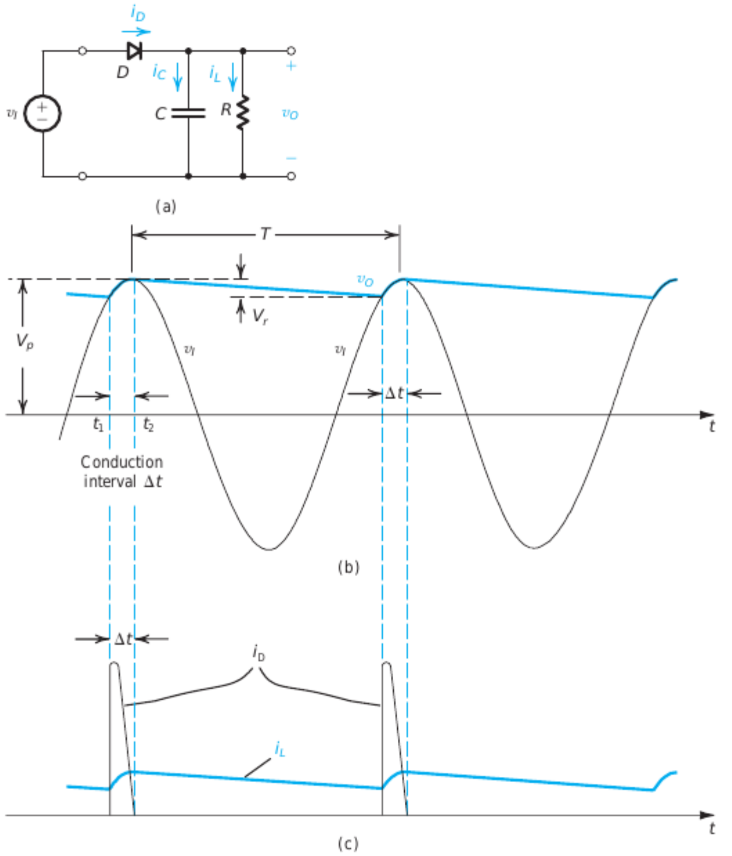
    - the capacitor will continue discharging, (for almost the entire cycle),
        - until the time at which v_I exceeds the capacitor voltage.
        - at this time, diode becomes forward-biased and
        - charges the capacitor upto the peak of v_I
        - and the process repeats itself.
- Note that to keep the output voltage from
    - decreasing too much during capacitor discharge,
    - select a value of C so that the time constant CR
        - is much greater than the discharge interval
        - i.e. $\tau = CR > T$ where discharge interval $\approx T$



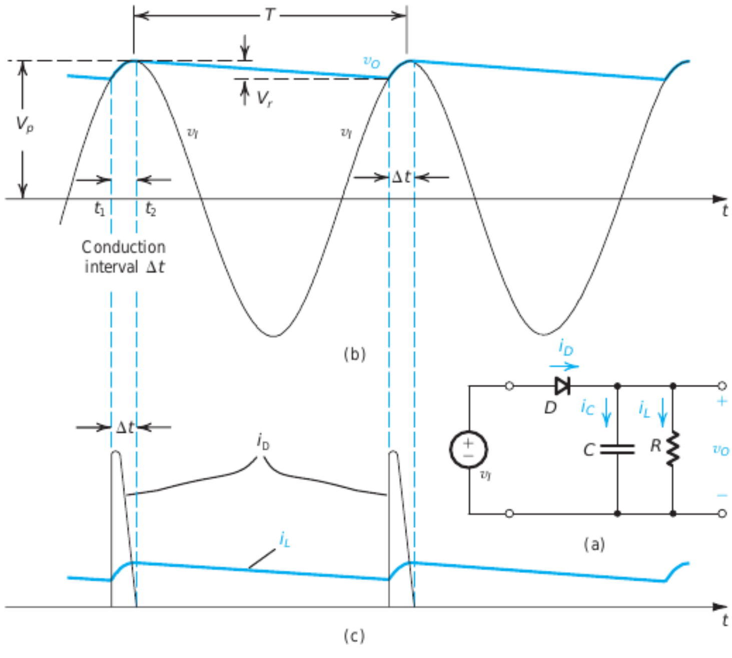
    - Output waveform of rectifier for different values of capacitor.
- 

    - the figure shows the steady state input and output voltage waveforms,
        - under the assumption $CR > T$, where T is the period of the input sinusoid
- the load current is $i_L = \frac{v_O}{R}$
    - the diode current (when diode is conducting) is

- $i_D = i_C + i_L$
- $i_D = C\frac{dv_I}{dt} + i_L$

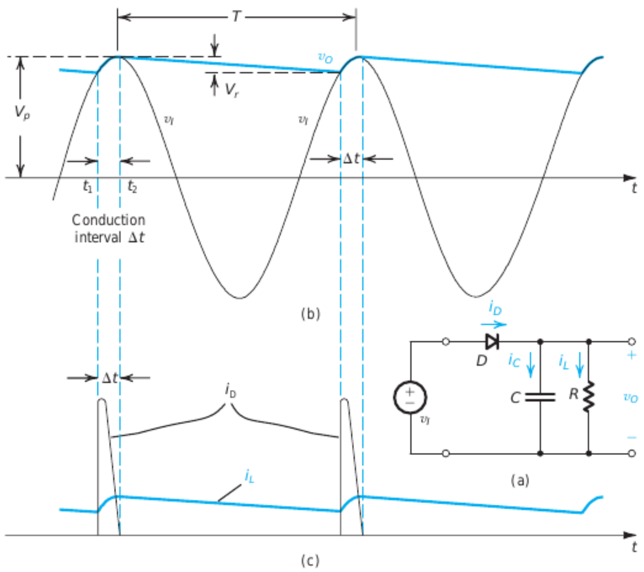


- **Figure 4.25** Voltage and current waveforms in the peak rectifier circuit with $CR \gg T$. The diode is assumed ideal.

- The diode conducts only for a brief interval Δt,
  - near the peak of the input sinusoid
    - the diode supplies the capacitor with charge equal to that lost during the discharge interval
    - the charging interval Δt is much smaller that the discharging interval $(\approx T)$
    - i.e. $\Delta t < T$
    - 

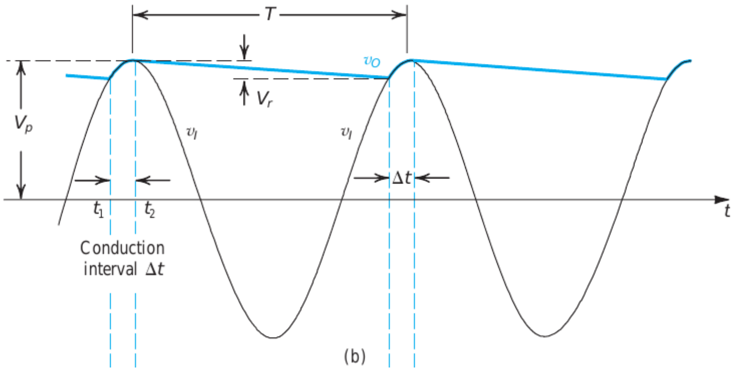
- the diode conduction begins at time t_1, when the input v_I
    - equals the exponentially decaying output v_O
        - the diode cuts off at time t_2, just after the peak of v_I
- during the diode off interval, the capacitor C
    - provides the load current. thus the capacitor discharges through R
        - and thus v_O decays exponentially with a time constant CR
    - the discharge interval starts just past the peak of v_I
    - the discharge interval lasts for almost the entire period
        - $T \approx T - \Delta t$
        - at the end of discharge interval $v_O = V_P - V_r$
            - where V_r is the peak to peak ripple voltage
            - if $CR > T$, the value of V_r is small
- If V_r is small, v_O is almost constant and can be approximated as
    - equal to the peak value of v_I
        - i.e. $v_O \approx V_P$
            - $= \Rightarrow i_L \approx \frac{V_P}{R}$

let's now derive expression for V_r and for average and peak diode currents

- during the diode off interval, v_O can be given as
    - $v_O = V_P e^{-t/CR}$
        - at the end of the discharge interval $(t = T - \Delta t$ and $v_O = V_P - V_r)$
            - $= \Rightarrow V_P - V_r = V_P e^{-(T - \Delta t)/CR}$
        - as Δt is small $= \Rightarrow T - \Delta t \approx T$
            - $V_P - V_r \approx V_P e^{-T/CR}$
        - 

- $V_P - V_r \approx V_P e^{-T/CR}$
  - as $CR > T = \Rightarrow \frac{T}{CR} < 1$
  - as $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$
    - $= \Rightarrow e^{-T/CR} = 1 - \frac{T}{CR} + \frac{1}{2!}\left(\frac{T}{CR}\right)^2 - \frac{1}{3!}\left(\frac{T}{CR}\right)^2 + \cdots$
  - for $\frac{T}{CR} < 1$, we can neglect 3rd term and higher order terms
    - $e^{-T/CR} \approx 1 - \frac{T}{CR}$
  - $= \Rightarrow V_P - V_r \approx V_P e^{-T/CR} \approx V_P\left(1 - \frac{T}{CR}\right)$
    - $= \Rightarrow V_P - V_r \approx V_P - \frac{T}{CR} V_P$
    - $-V_r \approx -\frac{T}{CR} V_P$ or $V_r \approx \frac{T}{CR} V_P$
  - $V_r \approx \frac{T}{CR} V_P$
  - $= \Rightarrow$ if $\frac{T}{CR}$ is small then V_r is small
  - As $T = \frac{1}{f} = \Rightarrow V_r \approx \frac{V_P}{fCR}$
- $V_r \approx \frac{T}{CR} V_P$
  - $V_r \approx \frac{V_P}{fCR}$
  - as $I_L = \frac{V_P}{R}$
    - $= \Rightarrow V_r = \frac{V_P}{fCR} = \frac{I_L}{fC}$

The conduction interval, Δt

- can be determined by noting that the
  - diode starts conducting at $t = t_1 = 0 - \Delta t = -\Delta t$ and
    - diode becomes reverse biased at the peak of v_I i.e. at $t = 0 = t_2$
    - 

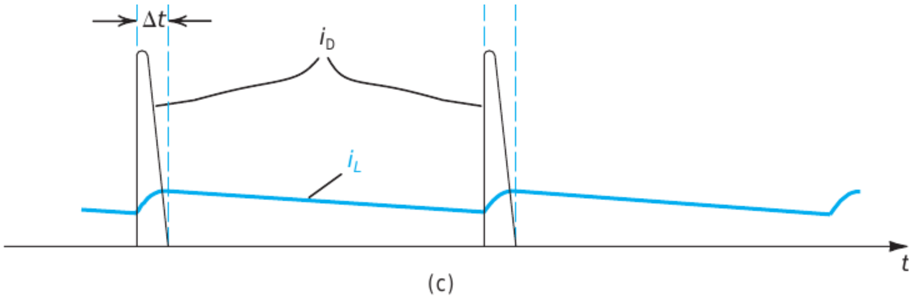
- from the figure
  - $V_P \cos(\omega\{-\Delta t\}) = (V_P - V_r)$
    - where $\omega = 2\pi f = \frac{2\pi}{T}$
    - As cos is an even function $= \Rightarrow \cos(-x) = \cos(x)$
    - $= \Rightarrow V_P \cos(\omega \Delta t) = (V_P - V_r)$

- $V_P \cos(\omega \Delta t) = (V_P - V_r)$
  - cosine can be represented in terms of series
    - $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$
    - as $\omega \Delta t$ is a small number, we can neglect 3rd and higher terms
      - $\cos\left(\omega \Delta t\right) \approx 1 - \frac{(\omega \Delta t)^2}{2!}$
    - $= \Rightarrow V_P \left(1 - \frac{(\omega \Delta t)^2}{2!}\right) \approx \left(V_P - V_r\right)$
      - or $V_P - V_P \frac{(\omega \Delta t)^2}{2!} \approx V_P - V_r$
      - $-V_P \frac{(\omega \Delta t)^2}{2!} \approx -V_r$
    - $(\omega \Delta t)^2 \approx \frac{2V_r}{V_P} = \Rightarrow \omega \Delta t \approx \sqrt{\frac{2V_r}{V_P}}$
    - $\omega \Delta t$ is the conduction angle

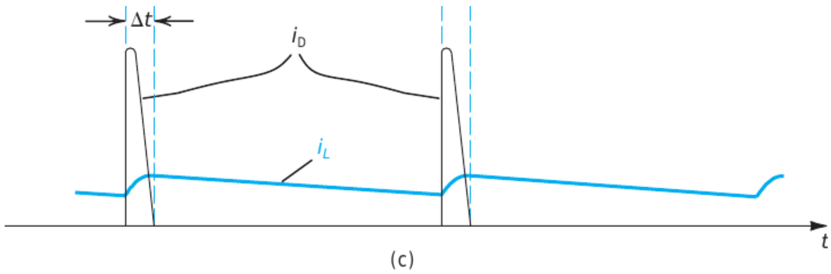
i_{Dav}:

- lets now determine, the average diode current during conduction, i_{Dav} by equating
  - the charge that the diode supplies to the capacitor, $Q_{supplied}$
    - to the charge that the capacitor losses during the discharge interval , Q_{lost}
      - i.e. $Q_{supplied} = Q_{lost}$
    - the charge supplied to the capacitor by the coducting diode can be given as $Q_{supplied} = i_{Cav} \Delta t$
    - As $i_D = i_C + i_L$, when diode is conducting
      - $i_{Dav} = i_{Cav} + I_L \because i_L \approx I_L = \frac{V_P}{R}$
      - $i_{Cav} = i_{Dav} - I_L$
    - $Q_{supplied} = i_{Cav} \Delta t = \left(i_{Dav} - I_L\right) \Delta t$
- $Q_{supplied} = i_{Cav} \Delta t = \left(i_{Dav} - I_L\right) \Delta t$
  - As $\omega \Delta t = \sqrt{\frac{2V_r}{V_P}}$
    - $= \Rightarrow Q_{supplied} = \left(i_{Dav} - I_L\right) \frac{1}{\omega} \sqrt{\frac{2V_r}{V_P}}$
    - the charge lost by the capacitor during the discharge interval can be given as
      - $Q_{lost} = CV_r$
    - As $Q_{supplied} = Q_{lost}$
      - $= \Rightarrow \left(i_{Dav} - I_L\right) \frac{1}{\omega} \sqrt{\frac{2V_r}{V_P}} = CV_r$

- $i_{Dav} - I_L = CV_r\omega\sqrt{\frac{V_P}{2V_r}}$

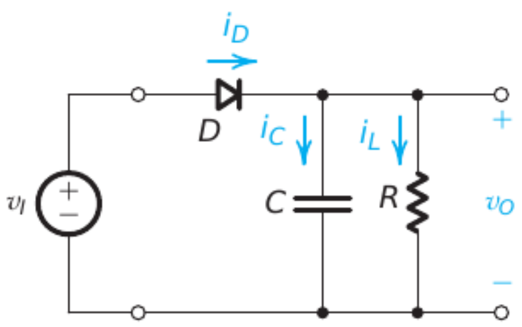
- $i_{Dav} - I_L = C\left(\frac{I_L}{fC}\right)\omega\sqrt{\frac{V_P}{2V_r}} \because V_r = \frac{I_L}{fC}$
  - $i_{Dav} - I_L = C\left(\frac{I_L}{fC}\right)2\pi f\sqrt{\frac{V_P}{2V_r}}$
    - $i_{Dav} - I_L = I_L 2\pi\sqrt{\frac{V_P}{2V_r}}$
    - $i_{Dav} = I_L + I_L 2\pi\sqrt{\frac{V_P}{2V_r}} = I_L\left(1 + 2\pi\sqrt{\frac{V_P}{2V_r}}\right)$
    - $i_{Dav} = I_L\left(1 + \pi\sqrt{\frac{2V_P}{V_r}}\right)$
- $i_{Dav} = I_L\left(1 + \pi\sqrt{\frac{2V_P}{V_r}}\right)$
  - Note that for $V_r < V_P$, the average diode current during conduction
    - is much greater than the dc load current
    - this is because the diode conducts for a very short duration.
    - 

i_{Dmax}:

- Peak value of the diode current i_{Dmax}, is its value at $t = t_1$ if t=0 is at the peak $= \Rightarrow t_1 = -\Delta t$
  - 

    - As $i_D = i_C + I_L$
      - $i_D\left(t\right) = C\frac{dv_I(t)}{dt} + I_L$ where $I_L = \frac{V_P}{R}$
    - $i_D\left(t\right) = C\frac{d}{dt}\left\{V_P\cos\omega t\right\} + I_L \because v_I(t) = V_P\cos\omega t$
    - $i_D(t) = CV_P\{-\sin\omega t\}(\omega) + I_L = \Rightarrow$ $i_D(t) = -C\omega V_P\sin\omega t + I_L$
- $i_D(t) = -C\omega V_P\sin\omega t + I_L$
  - i_D is maximum when $t = -\Delta t$

- $= \Rightarrow$ $i_{Dmax} = i_D(t = -\Delta t) = -C\omega V_P \sin\omega(-\Delta t) + I_L$
    - $= \Rightarrow i_{Dmax} = C\omega V_P \sin\omega\Delta t + I_L \because \sin(-x) = -\sin x$
- $i_{Dmax} = C\omega V_P \sin\omega\Delta t + I_L$
- As $\omega\Delta t \approx \sqrt{\frac{2V_r}{V_P}} < 1$ and As $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$
- $= \Rightarrow \sin\omega\Delta t = \omega\Delta t - \frac{(\omega\Delta t)^3}{3!} + \frac{(\omega\Delta t)^5}{5!} - \cdots$
- for $\omega\Delta t < 1 = \Rightarrow \sin\omega\Delta t \approx \omega\Delta t$
- $i_{Dmax} = C\omega V_P \sin\omega\Delta t + I_L = C\omega V_P \omega\Delta t + I_L$

- $i_{Dmax} = C\omega V_P \omega\Delta t + I_L$
    - using $\omega\Delta t \approx \sqrt{\frac{2V_r}{V_P}}$
        - $i_{Dmax} = C\omega V_P \sqrt{\frac{2V_r}{V_P}} + I_L$
            - $i_{Dmax} = C\omega\sqrt{2V_P V_r} + I_L$
            - $i_{Dmax} = I_L + C\omega\sqrt{2V_P V_r}$
            - $i_{Dmax} = I_L\left(1 + \frac{C\omega}{I_L}\sqrt{2V_P V_r}\right)$
        - using $V_r = \frac{I_L}{fC}$
            - $i_{Dmax} = I_L\left(1 + \frac{C\omega}{fCV_r}\sqrt{2V_P V_r}\right)$
            - $i_{Dmax} = I_L\left(1 + 2\pi\sqrt{\frac{2V_P}{V_r}}\right)$
- $i_{Dmax} = I_L\left(1 + 2\pi\sqrt{\frac{2V_P}{V_r}}\right)$ and $i_{Dav} = I_L\left(1 + \pi\sqrt{\frac{2V_P}{V_r}}\right)$
    - for $V_r < V_P$, the 2nd term $>$ 1st term for both i_{Dmax} and i_{Dav}
    - $= \Rightarrow i_{Dmax} \approx 2i_{Dav}$
- the peak rectifier that we studied till now is called a half-wave peak rectifier
    - 

    (a)
    - Similarly, the full wave rectifier circuits can also be
        - converted to peak rectifiers by including a capacitor across the load resistor

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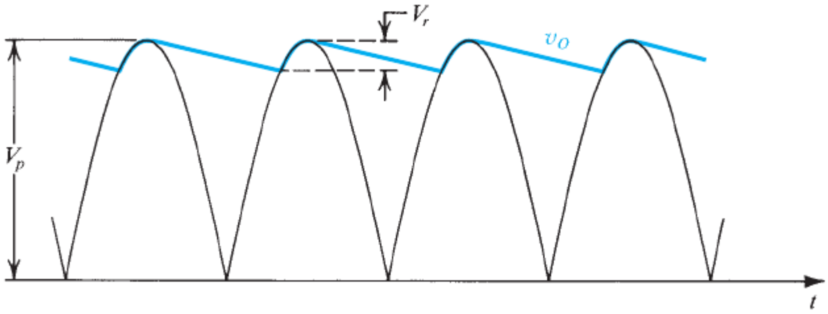
  - converted to peak rectifiers by including a capacitor across the load resistor



  - **Figure 4.26** Waveforms in the full-wave peak rectifier.
  - for a full wave peak rectifier,
    - the ripple frequency is twice that of the input
    - $= \Rightarrow$ the discharge period is $\frac{T}{2}$
  - The peak to peak ripple voltage, V_r can be obtained from the expression for half wave rectifier by repleacing T by $\frac{T}{2}$
- The peak to peak ripple voltage, V_r can be obtained from the expression for half wave rectifier by repleacing T by $\frac{T}{2}$
  - for half-wave peak rectifier
    - discharge period $\approx T$
    - $V_r = V_P \frac{T}{CR}$
    - $\omega \Delta t = \sqrt{\frac{2V_r}{V_P}}$
  - for full-wave peak rectifier
    - discharge period $\approx \frac{T}{2}$
    - $V_r = V_P \frac{T}{2CR}$
    - $\omega \Delta t = \sqrt{\frac{2V_r}{V_P}}$
  - Diode conduction interval Δt is the same for both half-wave and full-wave peak rectifiers
- for full wave peak rectifier, the average and peak diode currents can be given as
  - $i_{Dav} = I_L \left(1 + \pi \sqrt{\frac{V_P}{2V_r}}\right)$ and $i_{Dmax} = I_L \left(1 + 2\pi \sqrt{\frac{V_P}{2V_r}}\right)$
  - for half wave peak rectifier,
    - $i_{Dav} = I_L \left(1 + \pi \sqrt{\frac{2V_P}{V_r}}\right)$ and
      $i_{Dmax} = I_L \left(1 + 2\pi \sqrt{\frac{2V_P}{V_r}}\right)$
  - from these expressions of currents,
    - Note that for the same values of V_P , f , R and V_r,
    - a full-wave peak rectifier requires a capacitor half the size of that required in the half-wave peak rectifier

        - Also the current in each diode in the full-wave rectifier is approximately half that which flows in the diode of the half-wave circuit.
- In case of peak rectifiers, we have assumed ideal diodes
    - to improve the accuracy of above analysis,
        - one can take the diode voltage drop into account.
        - i.e. replace peak voltage V_P to which the capacitor charges
        - with $V_P - V_D$ for half wave and fullwave with center tapped transformer
        - and with $V_P - 2V_D$ for the bridge rectifier

Exercise 4.23

- Consider a bridge-rectifier circuit with a filter capacitor C placed across the load resistor R for the case in which the transformer secondary delivers a sinusiod of 12 V (rms) having a 60 Hz frequency and assuming $V_D = 0.8V$ and a load resistance $R = 100\Omega$. Find the value of C that result in a ripple voltage no larger that 1 V peak-to-peak. What is the dc voltage at the output? Find the load current. Find the diodes' conduction angle. Provide the average and peak diode currents. What is the peak reverse voltage across each diode? Specify the diode in terms of its peak current and its PIV.

Solution:

- bridge rectifier
    - $f = 60Hz$, $V_D = 0.8V$, $R = 100\Omega$, $V_r = 1V$, $C = ?$
    - $V_S = 12\sqrt{2} = 16.97V$, where V_S is peak voltage of transformer secondary
    - the peak voltage across capacitor $V_P = V_S - 2V_D = 15.371V$
    - as above relations were derived by using ideal diode model. but the constant voltage drop of 0.8V can be accomodated by using $V_P = V_S - 2V_D = 15.371V$ in all the relations.
- as $V_r = \frac{V_P}{2fCR}$, here V_P is the peak voltage across capacitor = 15.371V
    - $= \Rightarrow C = \frac{V_P}{2fV_rR} = \frac{15.371}{2\times60\times1\times100} = 1280.9\mu F$
    - $I_L = \frac{V_P}{R} = \frac{15.371}{100} = 0.1537A$
- Conduction angle $= \omega\Delta t = \sqrt{\frac{2V_r}{V_P}} = \sqrt{\frac{2\times1}{15.371}} = 0.36rad = 20.667^o$
    - $i_{Dav} = I_L\left(1 + \pi\sqrt{\frac{V_P}{2V_r}}\right) = 0.15\left(1 + \pi\sqrt{\frac{15.371}{2\times1}}\right) = 1.4564A$
    - $i_{Dmax} = I_L\left(1 + 2\pi\sqrt{\frac{V_P}{2V_r}}\right) = 0.15\left(1 + 2\pi\sqrt{\frac{15.371}{2\times1}}\right) = 2.7628A$
- $PIV = V_S - V_D = 16.971 - 0.8 = 16.171V$

Limiter Circuits

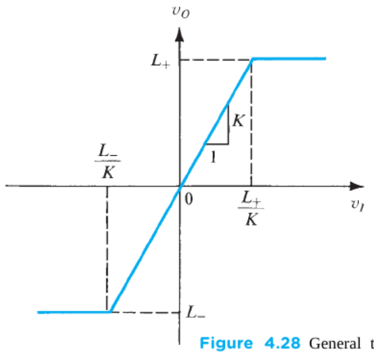
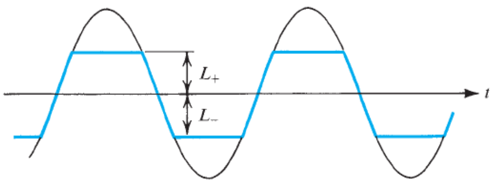
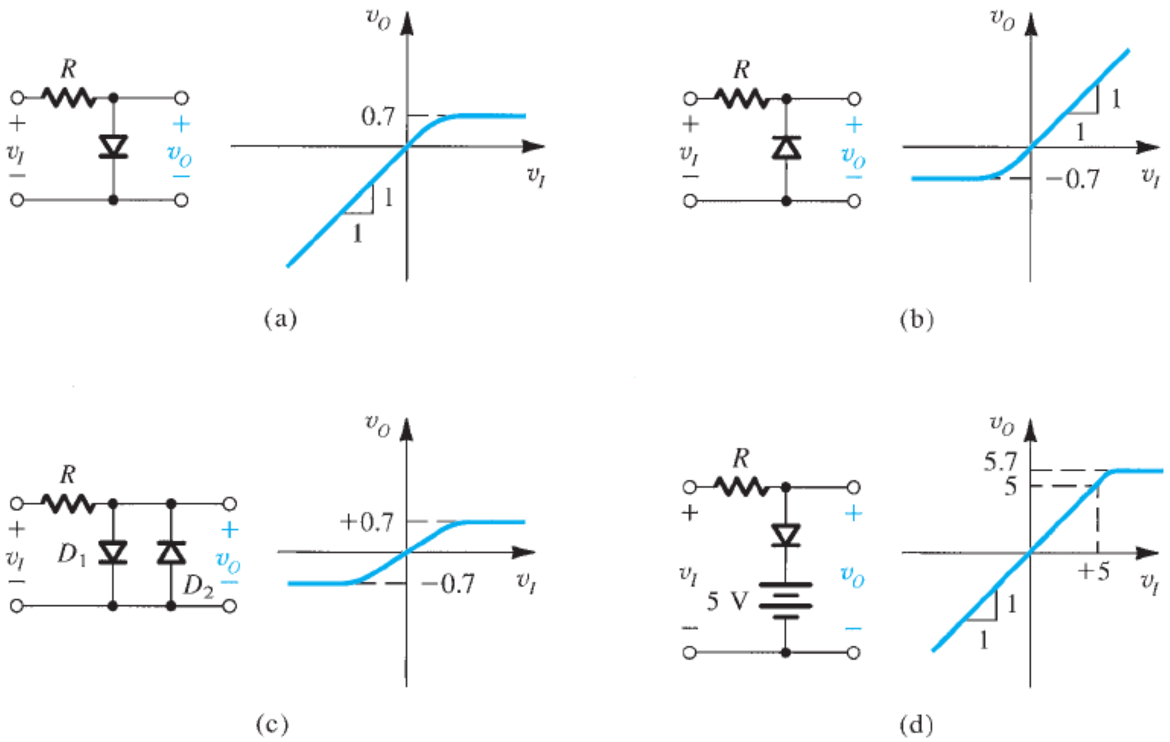
- general transfer characteristic of a limiter circuit can be given as
  - 

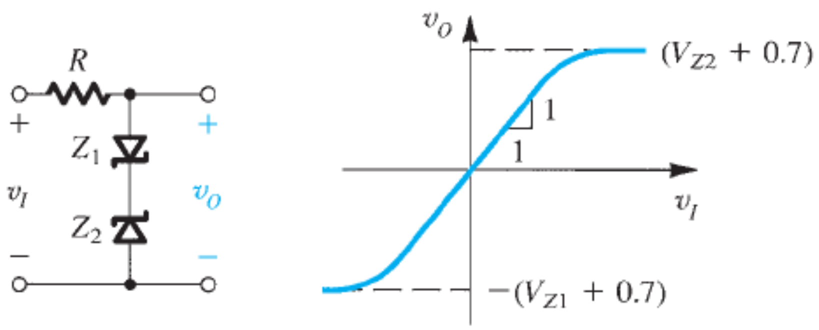
Figure 4.28 General transfer characteristic for a limiter circuit.
  - for input voltage v_I in a certain range of $\frac{L_-}{K} \leq v_I \leq \frac{L_+}{K}$
    - the limiter provides an output proportional to the input
    - i.e. $v_O = Kv_I$
  - if v_I exceeds the upper threshold $\frac{L_+}{K}$,
    - the output voltage is limited to the upper limiting value of L_+
- if v_I is reduced below the lower limiting threshold of $\frac{L_-}{K}$,
  - the output voltage is limited to the lower limiting value of L_-
  - A limiter that works on both the positive and negative peaks of an input waveform is called a double limiter
  - A single limiter only works on one of the peaks



- **Figure 4.29** Applying a sine wave to a limiter can result in clipping off its two peaks.
  - when the input waveform is fed to the double limiter,
    - its two peaks will be clipped off at the output
    - Limiters are therefore also called clippers


R
+
v_I
−
+
v_O
−
v_O
0.7
1
1
v_I
(a)
R
+
v_I
−
+
v_O
−
v_O
1
1
−0.7
v_I
(b)
R
D_1
D_2
v_O
+0.7
−0.7
v_I
(c)
R
5 V
v_O
5.7
5
+5
1
1
v_I
(d)

-


R
Z_1
Z_2
+
v_I
−
+
v_O
−
v_O
$(V_{Z2} + 0.7)$
1
1
v_I
$-(V_{Z1} + 0.7)$

(e)

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