Lecture 6b

EE-215 Electronic Devices and Circuits

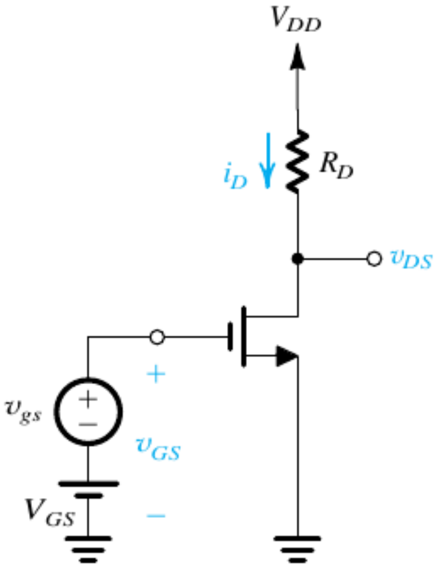
Asst Prof Muhammad Anis Chaudhary

Small-Signal Operation and Models

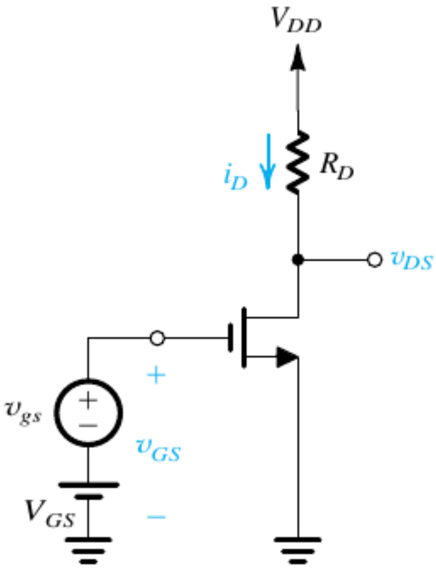
- Thus for MOSFET amplifier, linear amplification can be obtained by
  - biasing the MOSFET to operate in the saturation region
  - and by keeping the input signal small

The DC Bias Point

- for the conceptual amplifier circuit shown in fig
  - here the MOSFET is biased by applying a dc voltage V_{GS}
    - and the input signal to be amplified v_{gs} is superimposed on the dc bias voltage V_{GS}
    - the output voltage is v_{DS}
    - the dc bias current I_D can be determined by setting ac signal $v_{gs} = 0$
      - $= \Rightarrow I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2 = \frac{1}{2} k_n (V_{OV})^2$ if $\lambda = 0$

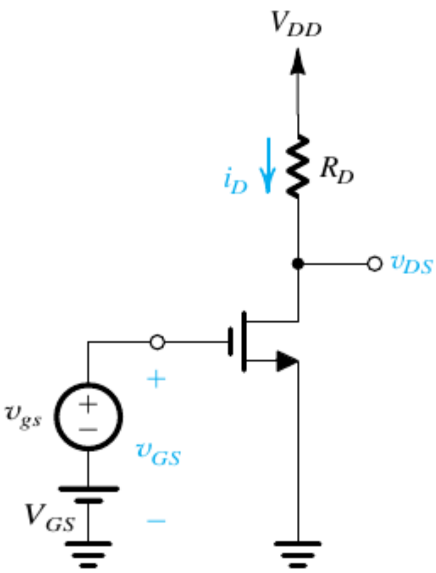


- the dc voltage at the drain can be given as (using KVL)
  - $V_{DD} = I_D R_D + V_{DS}$
    - $V_{DS} = V_{DD} - I_D R_D$
    - to ensure saturation region of operation
      - $V_{DS} > V_{OV}$
    - As the total voltage at the drain will have a signal component superimposed on V_{DS}
      - $= \Rightarrow V_{DS}$ has to be sufficiently greater than V_{OV} to
      - allow for the required signal swing



The Signal Current in the Drain Terminal

- Now if v_{gs} is applied, the total instantaneous gate-to-source voltage is
  - $v_{GS} = V_{GS} + v_{gs}$
    - and the total instantaneous drain current i_D is
      - $i_D = \frac{1}{2} k_n' \frac{W}{L} (v_{GS} - V_t)^2$
      - $i_D = \frac{1}{2} k_n (v_{GS} - V_t)^2$
      - $i_D = \frac{1}{2} k_n (V_{GS} + v_{gs} - V_t)^2$
      - $i_D = \frac{1}{2} k_n [(V_{GS} - V_t) + v_{gs}]^2$
      - $i_D = \frac{1}{2} k_n \left[(V_{GS} - V_t)^2 + 2\left(V_{GS} - V_t\right) v_{gs} + v_{gs}^2 \right]$



  - $i_D = \frac{1}{2} k_n (V_{GS} - V_t)^2 + k_n \left(V_{GS} - V_t\right) v_{gs} + \frac{1}{2} k_n v_{gs}^2$
- $i_D = \frac{1}{2} k_n (V_{GS} - V_t)^2 + k_n \left(V_{GS} - V_t\right) v_{gs} + \frac{1}{2} k_n v_{gs}^2$
  - the first term is the dc bias current i.e. $I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$
    - $= \Rightarrow i_D = I_D + k_n \left(V_{GS} - V_t\right) v_{gs} + \frac{1}{2} k_n v_{gs}^2$
  - the 2nd term of R.H.S represent a current component
    - that is directly proportional to v_{gs}

- the 3rd term is a current component that is proportional
    - to the square of the input signal.
    - this 3rd term represents non-linear distortion and is undesireable
- this non-linear distortion introduced by MOSFET can be
    - reduced by keeping the input sinal small
    - i.e. 3rd term < 2nd term $\Longrightarrow \frac{1}{2}k_n v_{gs}^2 < k_n\left(V_{GS} - V_t\right)v_{gs}$

- $\frac{1}{2}k_n v_{gs}^2 < k_n\left(V_{GS} - V_t\right)v_{gs}$
    - $\Longrightarrow \frac{1}{2}v_{gs}^2 < \left(V_{GS} - V_t\right)v_{gs}$
        - $\Longrightarrow v_{gs} < 2(V_{GS} - V_t)$
        - or $v_{gs} < 2V_{OV}$
        - this is the small-signal condition
    - If this condition is satisfied, we can ignore the 3rd term in the expression of i_D
    - $\Longrightarrow i_D \approx I_D + k_n(V_{GS} - V_t)v_{gs}$
    - or $i_D \approx I_D + i_d$
        - where $i_d = k_n(V_{GS} - V_t)v_{gs}$ or $i_d = g_m v_{gs}$, $g_m = k_n(V_{GS} - V_t)$
    - thus $g_m = k_n\left(V_{GS} - V_t\right) = k_n V_{OV} = \frac{i_d}{v_{gs}}$
        - this parameter that relates the i_d and v_{gs} is called the MOSFET transconductance
- Graphical interpretation of the small signal operation of the MOSFET amplifier is as shown in figure
    - Note that g_m is equal to the slope of the $i_D - v_{GS}$ characteristic at the bias point
        - i.e. $g_m = . \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{v_{GS}=V_{GS}}$
            - as $i_D = \frac{1}{2}k_n(v_{GS} - V_t)^2$
            - $g_m = . \left.\frac{\partial}{\partial v_{GS}}\left\{\frac{1}{2}k_n(v_{GS} - V_t)^2\right\}\right|_{v_{GS}=V_{GS}}$
            - $g_m = . \left.\frac{1}{2}k_n 2\left(v_{GS} - V_t\right)1\right|_{v_{GS}=V_{GS}}$
            - $g_m = k_n(V_{GS} - V_t)$

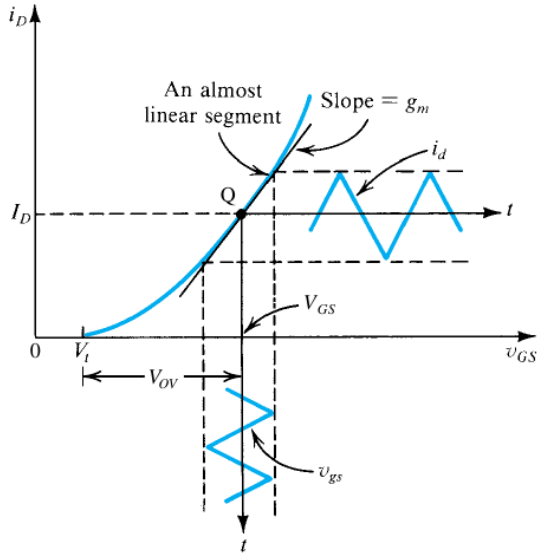


Figure 5.35 Small-signal operation of the MOSFET amplifier.

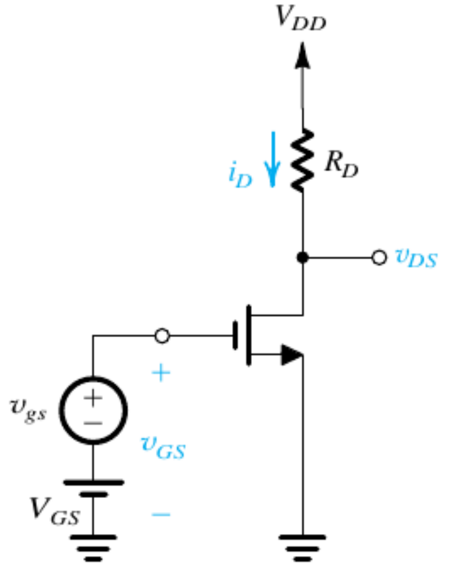
The Transconductance, g_m

- $g_m = k_n(V_{GS} - V_t) = k_n V_{OV}$
    - As
        - $I_D = \frac{1}{2}k_n{'}\frac{W}{L}(V_{GS} - V_t)^2 = \frac{1}{2}k_n V_{OV}^2$

        - $= \Rightarrow k_n V_{OV} = \frac{2I_D}{V_{OV}}$ or $V_{OV} = \sqrt{\frac{2I_D}{k_n}}$
    - $g_m = k_n V_{OV} = \frac{2I_D}{V_{OV}}$
    - $g_m = k_n V_{OV} = k_n \sqrt{\frac{2I_D}{k_n}} = \sqrt{k_n}\sqrt{2I_D} = \sqrt{2k_n'}\sqrt{\frac{W}{L}}\sqrt{I_D}$
    - $$g_m = k_n V_{OV} = \frac{2I_D}{V_{OV}} = \sqrt{2k_n'}\sqrt{\frac{W}{L}}\sqrt{I_D}$$

The Voltage Gain

- the total instantaneous drain voltage is
    - $v_{DS} = V_{DD} - R_D i_D$
        - under the small signal condition
            - $i_D = I_D + i_d$
        - $= \Rightarrow v_{DS} = V_{DD} - R_D i_D = V_{DD} - R_D(I_D + i_d)$
            - $v_{DS} = V_{DD} - R_D I_D - R_D i_d$
            - $v_{DS} = V_{DS} - R_D i_d \because V_{DS} = V_{DD} - R_D I_D$
        - $= \Rightarrow$ the signal component of v_{DS} is
            - $v_{ds} = -R_D i_d$
        - As $g_m = \frac{i_d}{v_{gs}} = \Rightarrow i_d = g_m v_{gs}$



- $v_{ds} = -R_D i_d$, $i_d = g_m v_{gs}$
    - $= \Rightarrow v_{ds} = -R_D i_d = -R_D(g_m v_{gs})$
        - $v_{ds} = -R_D g_m v_{gs}$ or $\frac{v_{ds}}{v_{gs}} = -R_D g_m$
        - $= \Rightarrow A_v = \frac{v_{ds}}{v_{gs}} = -R_D g_m$
        - here the minus sign indicates that the output signal is 180^o out of phase w.r.t. the input signal v_{gs}

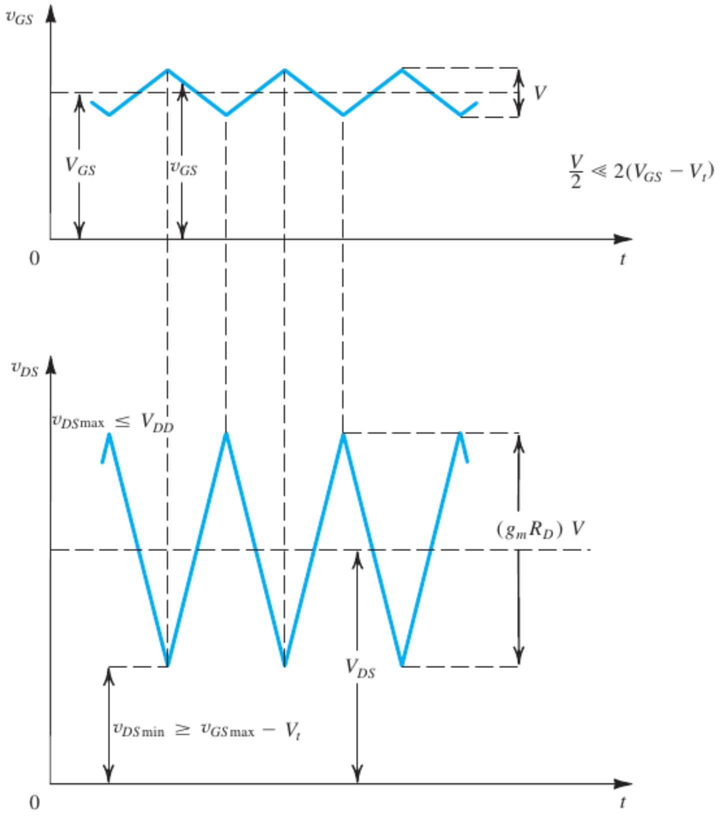


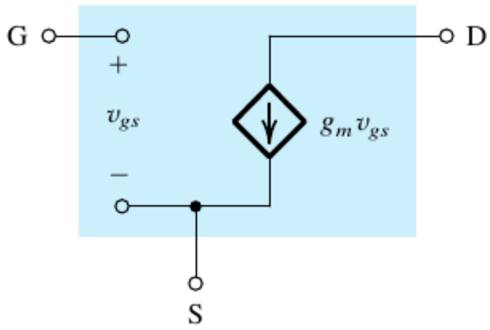
Figure 5.36 Total instantaneous voltages v_{GS} and v_{DS} for the circuit in Fig. 5.34.

Separating the DC Analysis and the Signal Analysis

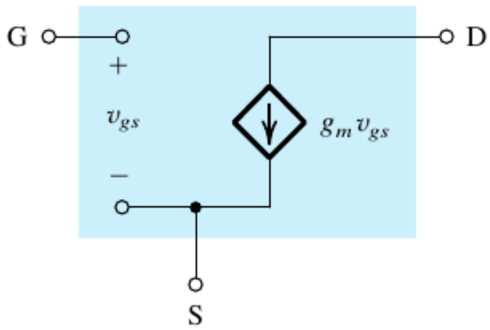
- thus under the small-signal approximation,
  - signal quatities are superimposed on dc quantities
    - $= \Rightarrow$ the total i_D equals the dc current I_D plus the signal current i_d
    - the total drain voltage v_{DS} equals $V_{DS} + v_{ds}$ and so on
  - thus the analysis and design can be greatly simplified ty
    - separating dc calculations from
    - small-signal or ac calculations
  - first we perform dc analysis by suppressing all ac sources
    - once dc operating point is determined, we proceed with ac analysis
  - for ac analysis, we suppress all dc sources,
    - i.e. dc voltage source is replaced by a short circuit
    - a dc current source is replaced by an open circuit

Small-Signal Equivalent Circuit Models

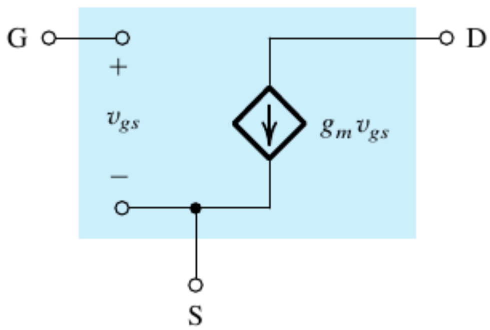
- we have already seen that under the small-signal condition
  - $i_d = g_m v_{gs}$ where $g_m = k_n(V_{GS} - V_t)$
    - $= \Rightarrow$ MOSFET behaves as a voltage controlled current source
    - it accepts a signal v_{gs} between gate and the source
      - and provides a current $g_m v_{gs}$ at the drain terminal
    - The input resistance at the gate is very high ideally infinite
    - the output resistance is also infinite (if $\lambda = 0$)
  - thus the small-signal operation of the MOSFET can be represented by figure



- this circuit shown in figure is called a small-signal model
  - this particular model is called the hybrid-π model
    - Thus in the analysis of a MOSFET amplifier circuit, we
      - first perform DC analysis
    - Once dc analysis is done,
      - replace the MOSFET by its small-signal model,
      - suppress the dc sources to obtain the circuit that
      - can be used to perform any required signal analysis



- the channel-length modulation effect can be
  - incorporated into the hybrid-π small-signal model
    - by placing the resistance r_o between drain and source
    - recall that $r_o = \frac{|V_A|}{I_D}$
      - where $V_A = \frac{1}{\lambda}$
      - recall that for a given process technology, $V_A \propto L$
      - I_D is value of dc drain current without the channel-length modulation taken into account
      - i.e. $I_D = \frac{1}{2} k_n V_{OV}^2 = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$



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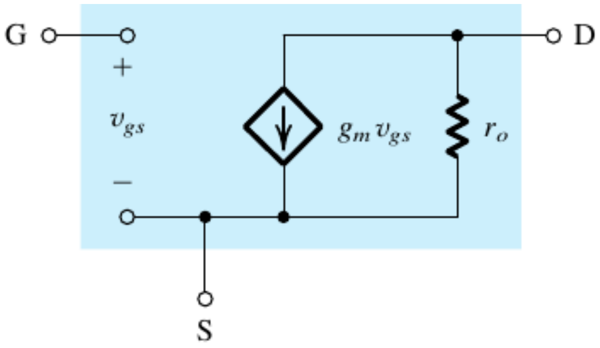
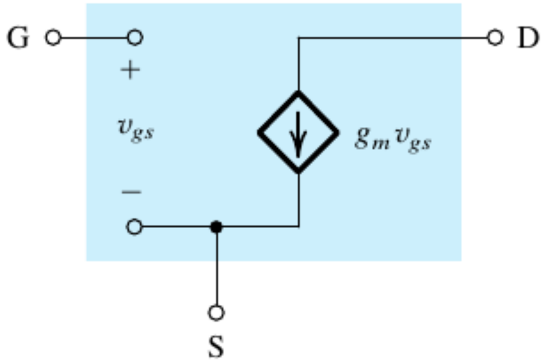
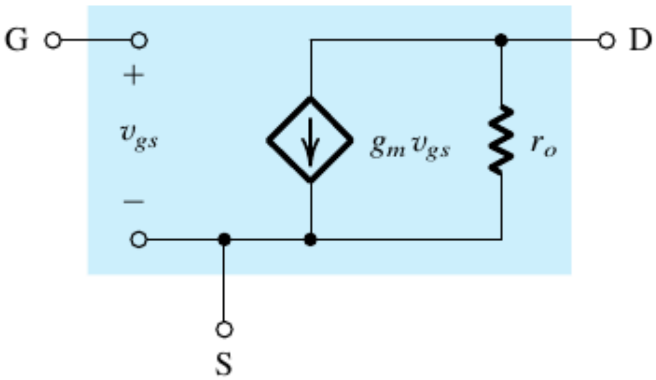


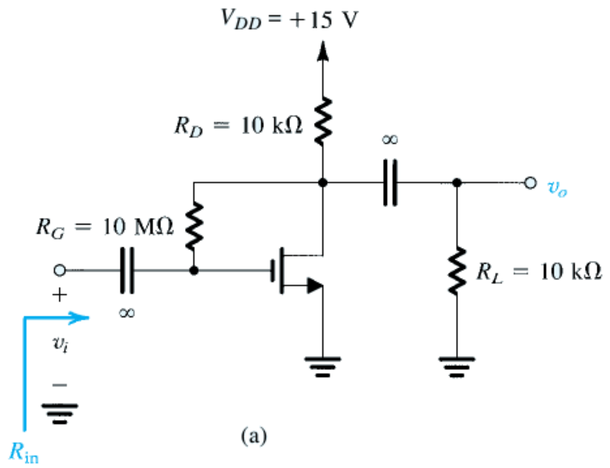
- Note that the small-signal model parameters g_m and r_o
  - depend on the dc bias point of the MOSFET
  - $g_m = k_n V_{OV} = \frac{2I_D}{V_{OV}} = \sqrt{2k_n'}\sqrt{\frac{W}{L}}\sqrt{I_D}$
  - $r_o = \frac{|V_A|}{I_D}$



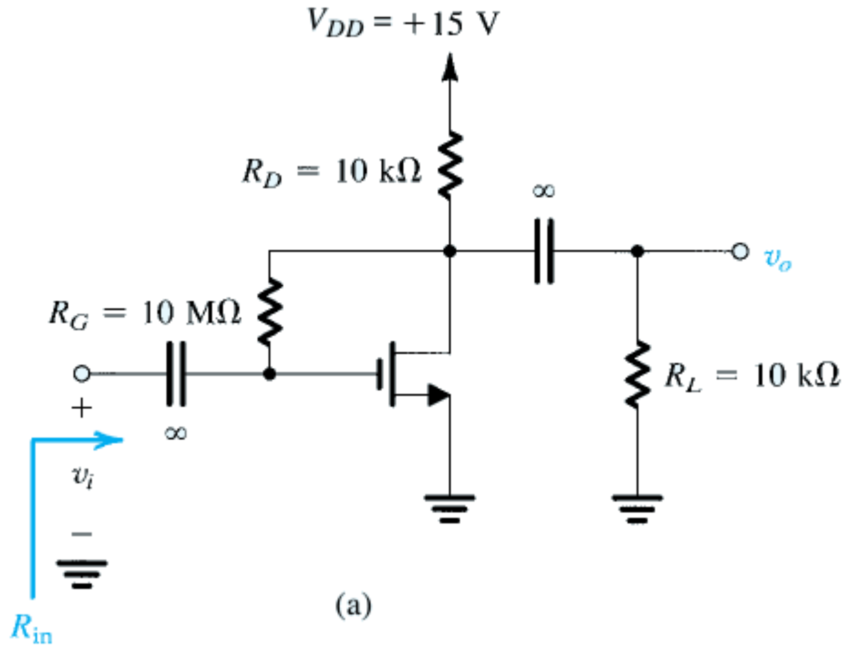


Example 5.10

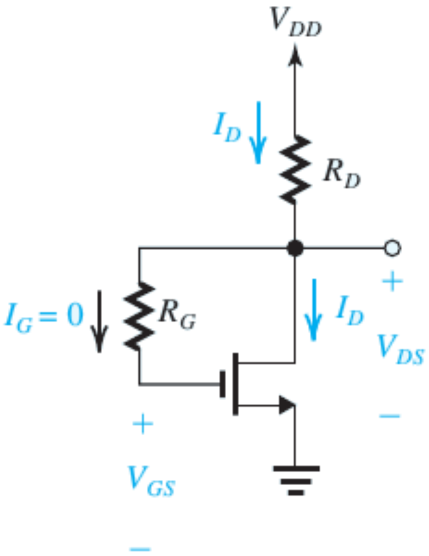
- Figure 5.39(a) shows a discrete common-source MOSFET amplifier utilizing a drain-to-gate resistance R_G for biasing purposes. Such a biasing arrangement will be studied in Section 5.7. The input signal v_i is coupled to the gate via a large capacitor, and the output signal at the drain is coupled to the load resistance R_L via another large capacitor. We wish to analyze this amplifier circuit to determine its small-signal voltage gain, its input resistance, and the largest allowable input signal. The transistor has $V_t = 1.5V$, $k_n'(W/L) = 0.25mA/V^2$, and $V_A = 50V$. Assume the coupling capacitors to be sufficiently large so as to act as short circuits at the signal frequencies of interest.



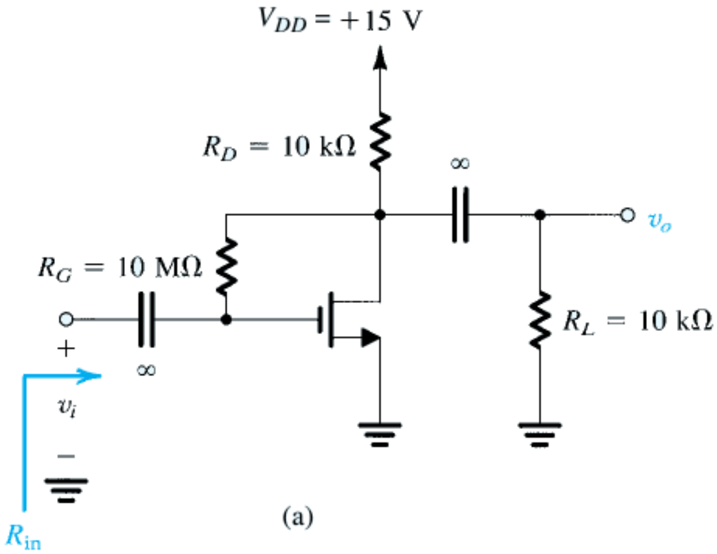
- Solution:
- $V_t = 1.5V$, $k_n'(W/L) = 0.25mA/V^2$, and $V_A = 50V$



  - first determine the dc operating point Q
- for this dc analysis
  - suppress ac signal sources
    - replace the two coupling capacitors by open circuits
      - (as these capacitors block dc currents)

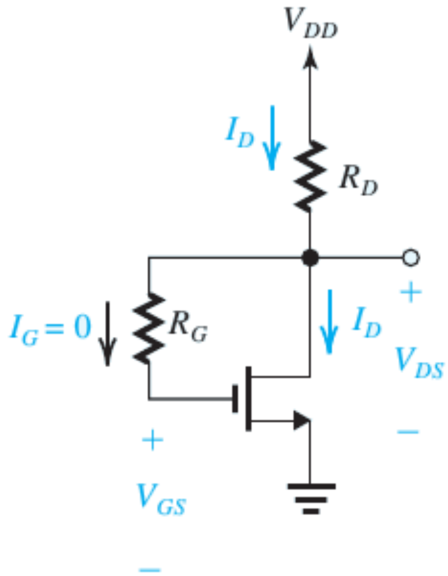


(b)



(a)

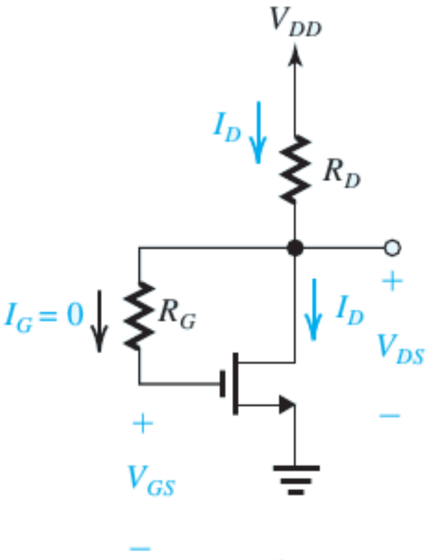
$= \Rightarrow$



(b)

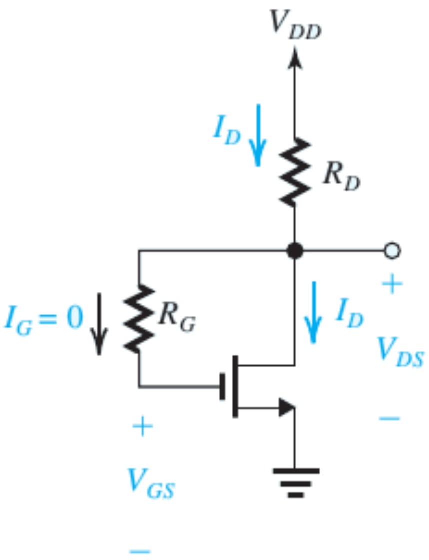
Example 5.10, DC Analysis

- here $I_G = 0 = \Rightarrow I_G R_G = 0$
  - $= \Rightarrow V_G = V_D$
    - or $V_{GS} = V_{DS}$ as source is grounded
    - by KVL
      - $V_{DD} = I_D R_D + V_{DS}$
      - $= \Rightarrow V_{GS} = V_{DS} = V_{DD} - I_D R_D = 15 - I_D(10k)$
    - for saturation
      - $V_{DS} \geq V_{GS} - V_t$
      - $V_{DS} - V_{GS} \geq -V_t$
      - $0 \geq -V_t \because V_{GS} = V_{DS}$
      - $0 \geq -1.5$ which is true $= \Rightarrow$MOSFET is in saturation



(b)

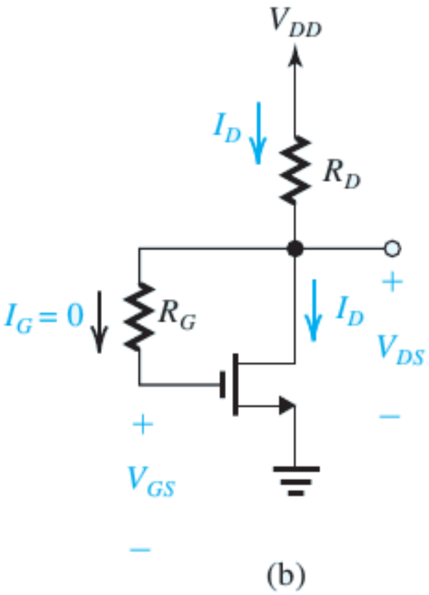
- neglecting the channel-length modulation effect on dc operating point
  - $= \Rightarrow I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$
    - $I_D = \frac{1}{2} \left(0.25m\right) (V_{GS} - 1.5)^2$
      - $I_D = 0.125m(V_{GS} - 1.5)^2$
    - As $V_{GS} = 15 - I_D(10k)$
      - $= \Rightarrow I_D = \frac{15 - V_{GS}}{10k}$
    - thus $I_D = \frac{15 - V_{GS}}{10k} = 0.125m(V_{GS} - 1.5)^2$
      - $\frac{15 - V_{GS}}{10k} = 0.125m(V_{GS} - 1.5)^2$
      - $15 - V_{GS} = 1.25(V_{GS} - 1.5)^2$



(b)

  - $15 - V_{GS} = 1.25\left(V_{GS}^2 + 2.25 - 3V_{GS}\right)$
- $15 - V_{GS} = 1.25\left(V_{GS}^2 + 2.25 - 3V_{GS}\right)$
  - $15 - V_{GS} = 1.25V_{GS}^2 + 2.8125 - 3.75V_{GS}$
    - $1.25V_{GS}^2 - 2.75V_{GS} - 12.188 = 0$

- $V_{GS} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- $V_{GS} = \frac{2.75 \pm \sqrt{(2.75)^2 - 4(1.25)(-12.188)}}{2(1.25)}$
- $V_{GS} = \frac{2.75 \pm 8.2766}{2.5} = 4.4106, -2.2106$
- -ve V_{GS} is not valid as the MOSFET is in saturation region
- $= \Rightarrow V_{GS} = 4.4106V$
- $V_{DS} = V_{GS} = 4.4106V$
- $I_D = \frac{15 - V_{GS}}{10k} = \frac{15 - 4.4106}{10k} = 1.0589mA$

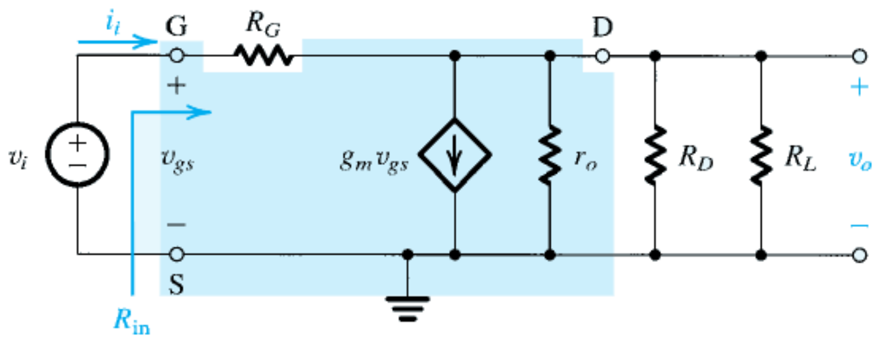


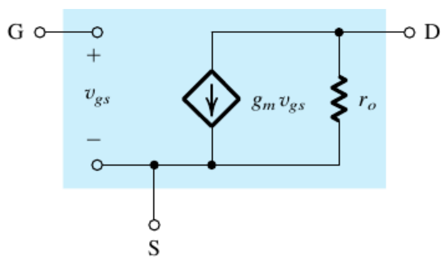
(b)

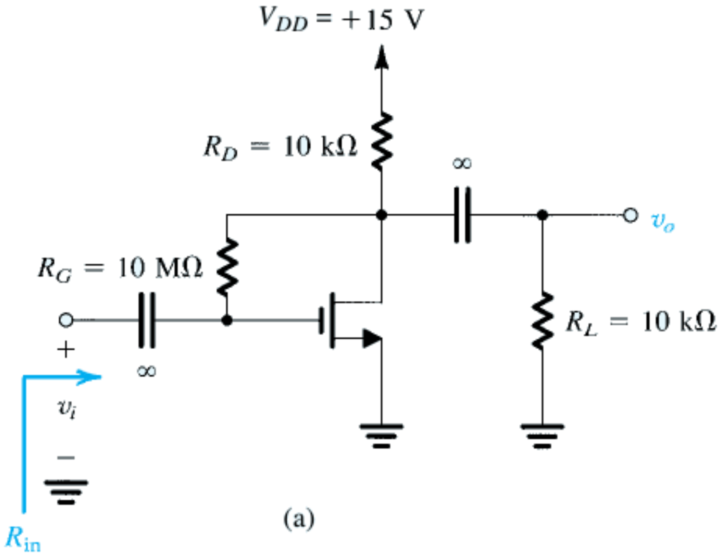
- $V_{OV} = V_{GS} - V_t = 4.4106 - 1.5 = 2.9106V$

Example 5.10, Small-signal Analysis

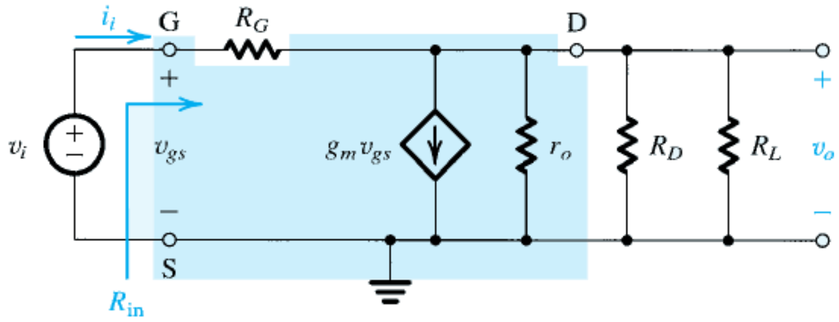
- Small-signal Analysis
  - Replace the MOSFET with its small-signal model
    - suppress the DC sources (replace V_{DD} by short-circuit)
      - replace coupling capacitors with short circuits

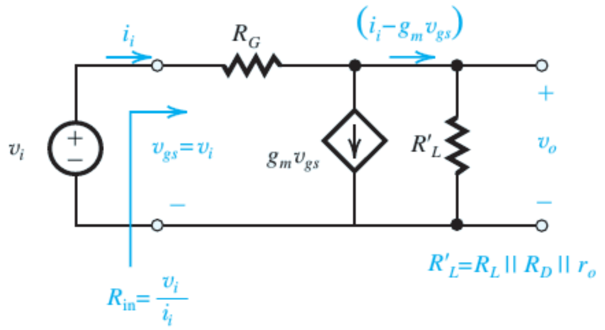




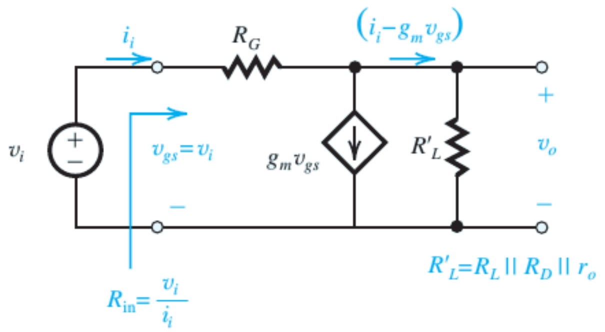


- as r_o , R_L and R_D are in parallel
  - $= \Rightarrow R_L' = r_o \parallel R_L \parallel R_D$





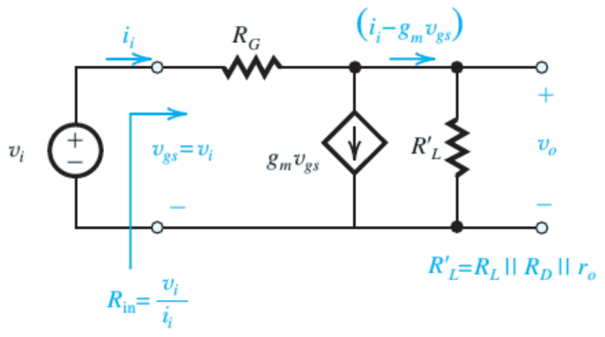
- $R_L' = r_o \parallel R_L \parallel R_D$



  - $r_o = \frac{V_A}{I_D} = \frac{50}{1.0589m} = 47.219k\Omega$
  - $R_L' = r_o \parallel R_L \parallel R_D = \left(\frac{1}{r_o} + \frac{1}{R_L} + \frac{1}{R_D}\right)^{-1} = \left(\frac{1}{47.219k} + \frac{1}{10k} + \frac{1}{10k}\right)^{-1} = 4.52k\Omega$

- $g_m = k_n V_{OV} = (0.25m)(2.9106) = 0.72765m\frac{A}{V}$
- input resistance, $R_{in} = \frac{v_i}{i_i}$

- input resistance, $R_{in} = \frac{v_i}{i_i}$



  - by KCL , $i_i = g_m v_{gs} + i_{R_L'} = g_m v_{gs} + \frac{v_o}{R_L'} = g_m v_{gs} + \frac{v_o}{4.52k}$
  - ohms' law at $R_G = \Rightarrow i_i R_G = v_i - v_o$
    - $= \Rightarrow i_i = \frac{v_i - v_o}{R_G}$
  - thus $i_i = \frac{v_i - v_o}{R_G} = g_m v_{gs} + \frac{v_o}{4.52k}$
    - $\frac{v_{gs} - v_o}{R_G} = g_m v_{gs} + \frac{v_o}{4.52k}$
- $\frac{v_{gs} - v_o}{R_G} = g_m v_{gs} + \frac{v_o}{4.52k}$
  - $\frac{v_{gs}}{R_G} - \frac{v_o}{R_G} = g_m v_{gs} + \frac{v_o}{4.52k}$
    - $\frac{v_{gs}}{R_G} - g_m v_{gs} = \frac{v_o}{4.52k} + \frac{v_o}{R_G}$
    - $v_{gs}\left(\frac{1}{R_G} - g_m\right) = v_o\left(\frac{1}{4.52k} + \frac{1}{R_G}\right)$
    - $A_v = \frac{v_o}{v_i} = \frac{v_o}{v_{gs}} = \left(\frac{1}{R_G} - g_m\right) / \left(\frac{1}{4.52k} + \frac{1}{R_G}\right)$
    - $A_v = \left(\frac{1}{10M} - 0.728m\right) / \left(\frac{1}{4.52k} + \frac{1}{10M}\right) = -3.2741\frac{V}{V}$
  - As $A_v = \frac{v_o}{v_i} = -3.2741\frac{V}{V} = \Rightarrow v_o = A_v v_i = A_v v_{gs}$
  - as $R_{in} = \frac{v_i}{i_i} = \frac{v_{gs}}{i_i}$
    - where $i_i = \frac{v_i - v_o}{R_G}$
  - $= \Rightarrow R_{in} = \frac{v_{gs}}{i_i} = \frac{R_G v_{gs}}{v_i - v_o} = \frac{R_G v_{gs}}{v_{gs} - A_v v_{gs}} = \frac{R_G v_{gs}}{v_{gs}(1 - A_v)} = \frac{R_G}{1 - A_v}$
  - $R_{in} = \frac{R_G}{1 - A_v} = \frac{10M}{1 - (-3.2741)} = 2.3397M\Omega$
- to avoid cutoff region
  - $v_{GS} \geq V_t$
  - $v_{GS_{min}} \geq V_t$
  - $V_{GS} - \hat{v}_{gs} \geq V_t$
  - $4.4106 - \hat{v}_{gs} \geq 1.5$
  - $-\hat{v}_{gs} \geq 1.5 - 4.4106$
  - $-\hat{v}_{gs} \geq -2.9106$
  - $\hat{v}_{gs} \leq 2.9106$
- to avoid triode region
  - $v_{DS} \geq v_{GS} - V_t$
  - $v_{DS_{min}} \geq v_{GS_{max}} - V_t$
  - $V_{DS} - \hat{v}_{ds} \geq (V_{GS} + \hat{v}_{gs}) - V_t$
  - as $A_v = \frac{v_o}{v_i} = \frac{v_{ds}}{v_{gs}} = \Rightarrow \hat{v}_{ds} = |A_v|\hat{v}_{gs}$
  - $V_{DS} - |A_v|\hat{v}_{gs} \geq V_{GS} + \hat{v}_{gs} - V_t$
  - as $V_{DS} = V_{GS} = 4.4106V$, $V_t = 1.5V$

  - $-|A_v|\hat{v}_{gs} \geq \hat{v}_{gs} - V_t$
  - $-3.2741\hat{v}_{gs} \geq \hat{v}_{gs} - 1.5$
- thus to avoid cutoff region
  - $\hat{v}_{gs} \leq 2.9106$
- thus to avoid triode region
  - $1.5 \geq \hat{v}_{gs} + 3.2741\hat{v}_{gs}$
  - $1.5 \geq 4.2741\hat{v}_{gs}$
  - $\hat{v}_{gs} \leq \frac{1.5}{4.2741} = 0.35095V$
