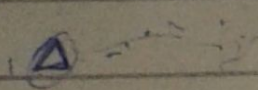


$$\frac{dz}{(z+1)(z-1)}$$

$$z_0 = 1$$

$$\frac{dz}{z+1}$$

$$f(1) = \frac{1}{2\pi i} \oint \frac{dz}{z+1}$$



$$\frac{1}{2} \times 2\pi i = \oint \frac{dz}{z+1}$$

∮

$$\oint \frac{dz}{z+1} = \pi i$$

### PROBLEM SET 14.4

1-8

Integrate counterclockwise around the circle  $|z| = 2$ . ( $n$  is a positive integer,  $a$  is arbitrary. Show details of your work.)

$$1 - \frac{\cos 3hz}{z^5} \quad C: |z|=2 \quad f(z) = \frac{\cos 3hz}{z^5} \quad n=4 \quad z_0=0$$

$$f^{(4)}(z) = \frac{4!}{2\pi i} \oint f(z) dz ;$$

$$-8! \cosh 3z - 4! \quad f(z) = \cos 3hz$$

(3) Alternating series test; suppose we have  
 $(-1)^n b_n$  or  $a_n = (-1)^{n+1} b_n$  where  $b_n \geq 0$ ,  
 if (a)  $\lim_{n \rightarrow \infty} b_n = 0$  and  $b_n$  is decreasing sequence, then series converges.

$$f\left(\frac{\pi}{2}\right) = \frac{1}{2\pi i} \oint f(z) dz$$

$$e^{\frac{\pi}{2}} \left( \frac{\pi}{2} (-1) \times 2\pi i - (-e^{\frac{\pi}{2}}) (\pi i) \right)$$

(3)  $e^z \cos z$

$$e^x (\cos y + i \sin y) (\cos x + iy)$$

$$(z - \frac{\pi}{2})^2$$

$$n=1$$

$$f'(z) = -e^z \sin z + z e^z \cos z$$

$$f'\left(\frac{\pi}{2}\right) = \frac{1}{2\pi i} \oint e^z \cos z$$

$$-e^{\frac{\pi}{2}} + \frac{\pi}{2} e^{\frac{\pi}{2}} = \frac{1}{2\pi i} \oint e^z \cos z$$

$$e^{\frac{\pi}{2}} \left( \frac{\pi}{2} - 1 \right) = \frac{1}{2\pi i} \oint e^z \cos z$$

$$2\pi i e^{\frac{\pi}{2}} \left( \frac{\pi}{2} - 1 \right) = \oint e^z \cos z$$

### Sequence and Series

+ The divergent test;  
 If  $\lim_{n \rightarrow \infty} z_n \neq 0$ , then  $\sum z_n$  is dgt. (If a series is a convergent, then  $\lim_{n \rightarrow \infty} z_n = 0$ , if it doesn't hold the series diverges)  
 If  $z_n = 0$ , the test doesn't hold.

Exp 1;  $\sum_1^{\infty} \left(1 + \frac{1}{n}\right)^n$

$$z_n = \left(1 + \frac{1}{n}\right)^n$$

$$\lim_{n \rightarrow \infty} z_n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$$= e \neq 0 \text{ dgt}$$

Exp 2;  $\sum_1^{\infty} \frac{\ln n}{1 + \ln(n)}$

$$z_n = \frac{\ln n}{1 + \ln n}$$

$$\Rightarrow \lim_{n \rightarrow \infty} z_n = \lim_{n \rightarrow \infty} \frac{\ln n}{1 + \ln n} = \frac{\infty}{\infty}$$

Apply L'Hopital's rule

$$\frac{\frac{1}{n}}{0 + \frac{1}{n}} \Rightarrow \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{\frac{1}{n}} = 1$$

Convergent.

Exp 3;  $\sum_1^{\infty} \frac{1}{n}$   $z_n = \frac{1}{n}$

$$\lim_{n \rightarrow \infty} z_n = \lim_{n \rightarrow \infty} \frac{1}{n} = 0 \text{ (divergent test not applicable)}$$