EE-232 Signals & Systems

Lecture 2

Signals Concepts & Properties

Asst Prof Kamran Aziz Bhatti

Signals Concepts & Properties

(1) Systems, **signals**, mathematical models.
Continuous-time and discrete-time signals.
Energy and power signals. Linear systems.
Examples for use throughout the course, introduction to Matlab and Simulink tools

Specific objectives for this lecture include

- General properties of signals
- Energy and power for continuous & discrete-time signals
- Signal transformations
- Specific signal types
- Representing signals in Matlab and Simulink

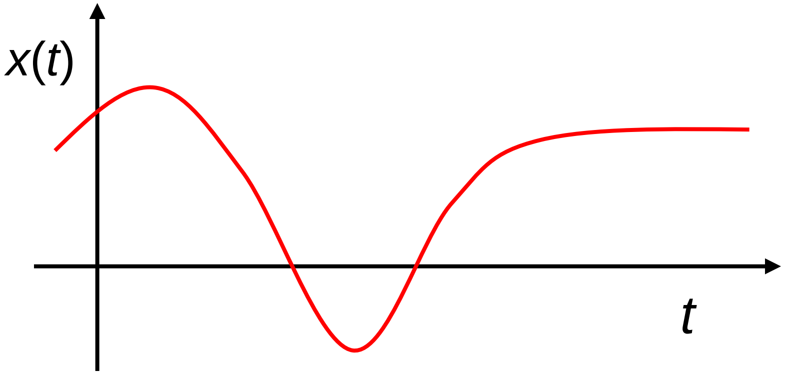
Reminder: Continuous & Discrete Signals

Continuous-Time Signals

Most signals in the real world are continuous time, as the scale is infinitesimally fine.

E.g. voltage, velocity,

Denote by $x(t)$, where the time interval may be bounded (finite) or infinite

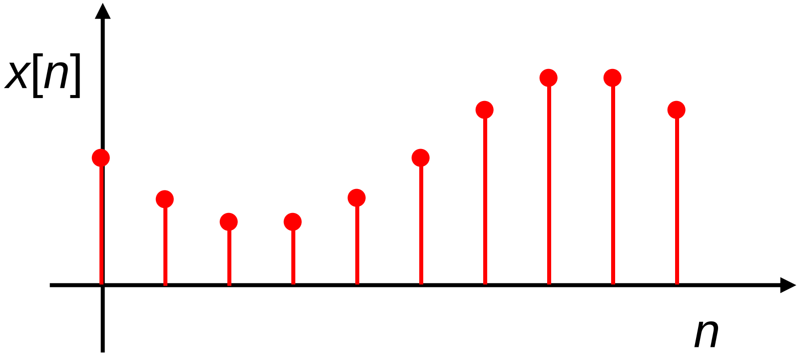


Discrete-Time Signals

Some real world and many digital signals are discrete time, as they are sampled

E.g. pixels, daily stock price (anything that a digital computer processes)

Denote by $x[n]$, where n is an integer value that varies discretely



Sampled continuous signal

$x[n] = x(nk)$

"Electrical" Signal Energy & Power

It is often useful to characterise signals by measures such as **energy** and **power**

For example, the **instantaneous power** of a resistor is:

$$p(t) = v(t)i(t) = \frac{1}{R}v^2(t)$$

and the **total energy** expanded over the interval [t_1, t_2] is:

$$\int_{t_1}^{t_2} p(t)dt = \int_{t_1}^{t_2} \frac{1}{R}v^2(t)dt$$

and the **average energy** is:

$$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)dt = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \frac{1}{R}v^2(t)dt$$

How are these concepts defined for any continuous or discrete time signal?

Generic Signal Energy and Power

Total energy of a continuous signal $x(t)$ over $[t_1, t_2]$ is:

$$E = \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

where |.| denote the magnitude of the (complex) number.

Similarly for a discrete time signal $x[n]$ over $[n_1, n_2]$:

$$E = \sum_{n=n_1}^{n_2} |x[n]|^2$$

By dividing the quantities by $(t_2 - t_1)$ and $(n_2 - n_1 + 1)$, respectively, gives the **average power**, P

Note that these are similar to the electrical analogies (voltage), but they are different, both value and dimension.

Energy and Power over Infinite Time

For many signals, we're interested in examining the power and energy over an infinite time interval (-∞, ∞). These quantities are therefore defined by:

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

If the sums or integrals do not converge, the energy of such a signal is infinite

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Two important (sub)classes of signals

1. Finite total energy (and therefore zero average power)
2. Finite average power (and therefore infinite total energy)

Signal analysis over infinite time, all depends on the "tails" (limiting behaviour)

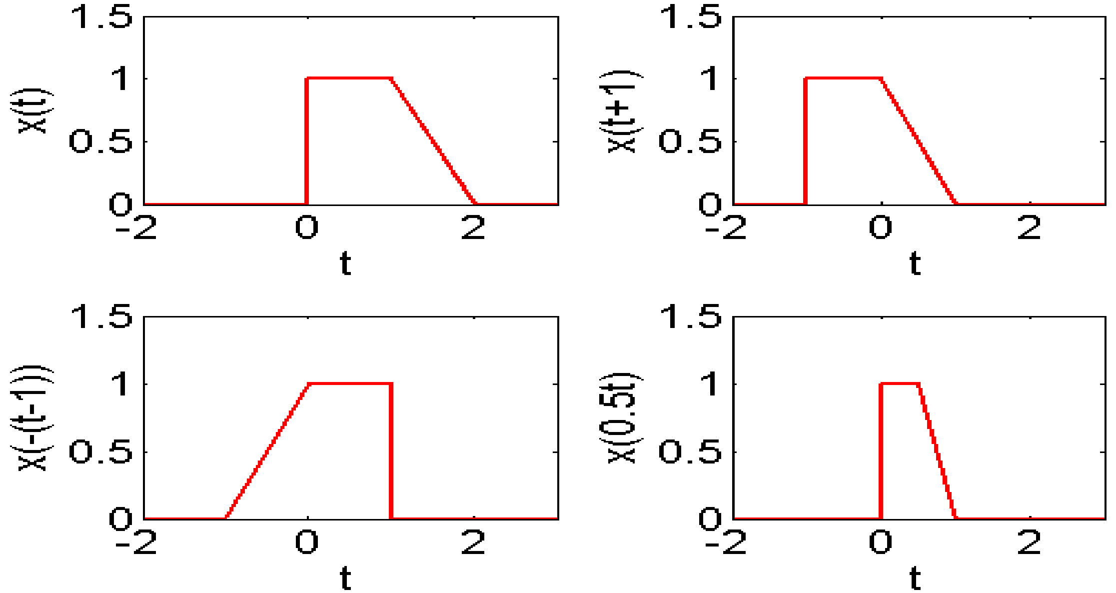
Time Shift Signal Transformations

A central concept in signal analysis is the transformation of one signal into another signal. Of particular interest are simple transformations that involve a transformation of the time axis only.

A linear **time shift** signal transformation is given by:

$$y(t) = x(at + b)$$

where *b* represents a signal offset from 0, and the a parameter represents a signal stretching if $|a|>1$, compression if $0<|a|<1$ and a reflection if $a<0$.

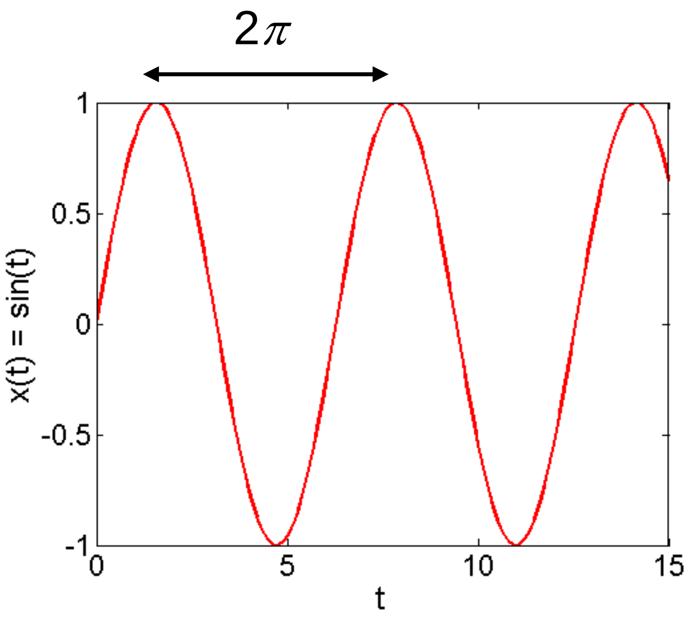


Periodic Signals

An important class of signals is the class of periodic signals. A periodic signal is a continuous time signal x(t), that has the property

$$x(t) = x(t+T)$$

where $T>0$, for all t.



Examples:

cos(t+2π) = cos(t)

sin(t+2π) = sin(t)

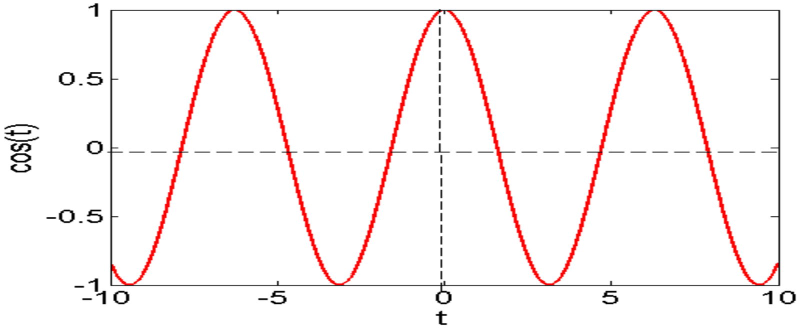
Are both periodic with period 2π

NB for a signal to be periodic, the relationship must hold for all t.

Odd and Even Signals

An **even** signal is identical to its time reversed signal, i.e. it can be reflected in the origin and is equal to the original:

$$x(-t) = x(t)$$



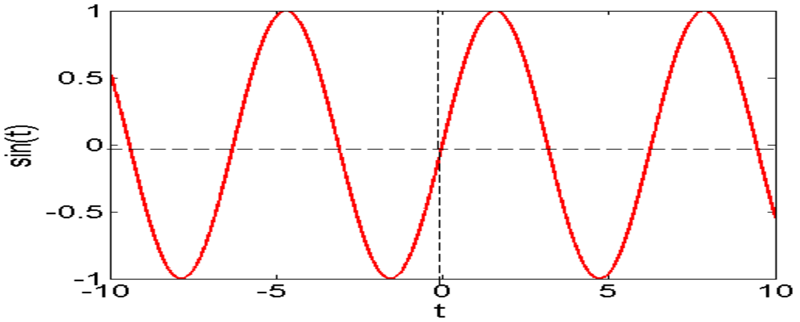
Examples:

x(t) = cos(t)

x(t) = c

An **odd** signal is identical to its negated, time reversed signal, i.e. it is equal to the negative reflected signal

$$x(-t) = -x(t)$$



Examples:

x(t) = sin(t)

x(t) = t

This is important because any signal can be expressed as the sum of an odd signal and an even signal.

Exponential and Sinusoidal Signals

Exponential and sinusoidal signals are characteristic of real-world signals and also from a basis (a building block) for other signals.

A generic **complex exponential signal** is of the form:

$$x(t) = Ce^{at}$$

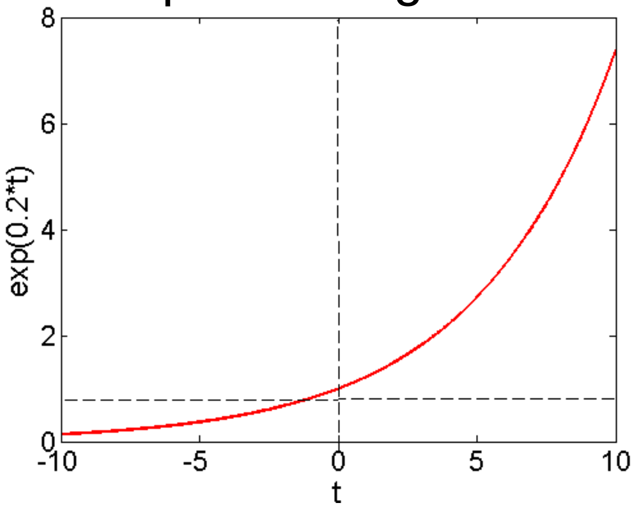
where *C* and *a* are, in general, complex numbers. Lets investigate some special cases of this signal

Real exponential signals

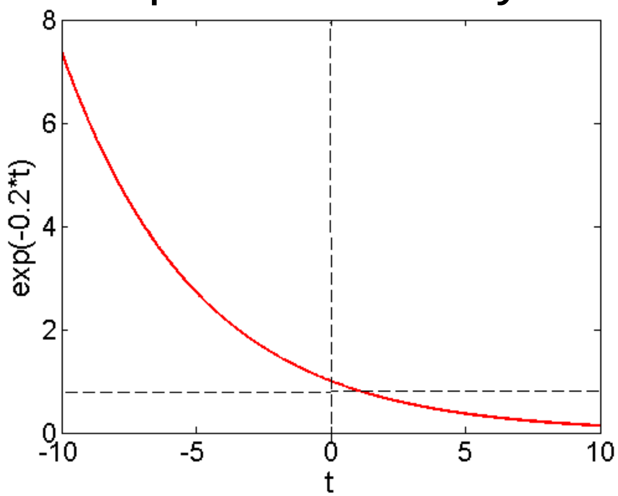
$a > 0$

$C > 0$

Exponential growth



Exponential decay



$a < 0$

$C > 0$

Periodic Complex Exponential & Sinusoidal Signals

Consider when *a* is purely imaginary:

$$x(t) = Ce^{j\omega_0 t}$$

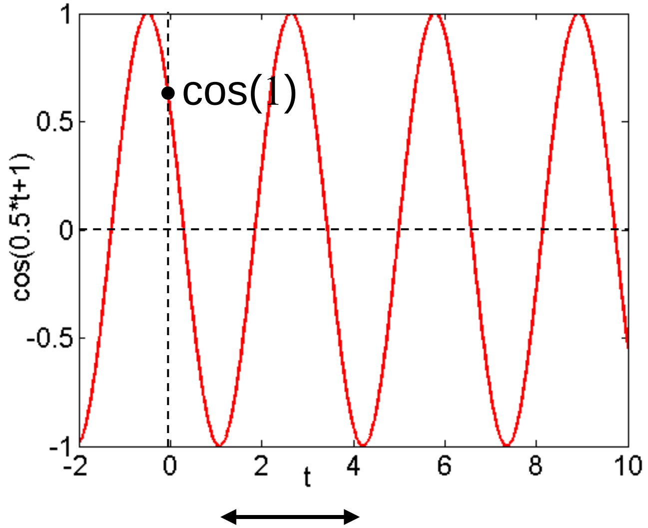
By Euler's relationship, this can be expressed as:

$$e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$$

This is a periodic signals because:

$$e^{j\omega_0 (t+T)} = \cos\omega_0 (t+T) + j\sin\omega_0 (t+T)$$
$$= \cos\omega_0 t + j\sin\omega_0 t = e^{j\omega_0 t}$$

when $T=2\pi/\omega_0$



$T_0 = 2\pi/\omega_0$
$= \pi$

A closely related signal is the **sinusoidal signal**:

$$x(t) = \cos(\omega_0 t + \phi) \qquad \omega_0 = 2\pi f_0$$

T_0 is the fundamental time period

ω_0 is the fundamental frequency

We can always use:

$$A\cos(\omega_0 t + \phi) = A\Re\left(e^{j(\omega_0 t+\phi)}\right)$$
$$A\sin(\omega_0 t + \phi) = A\Im\left(e^{j(\omega_0 t+\phi)}\right)$$

Exponential & Sinusoidal Signal Properties

Periodic signals, in particular complex periodic and sinusoidal signals, have infinite total energy but finite average power.

Consider energy over one period:

$$E_{period} = \int_0^{T_0} \left| e^{j\omega_0 t} \right|^2 dt$$

$$= \int_0^{T_0} 1 dt = T_0$$

Therefore:

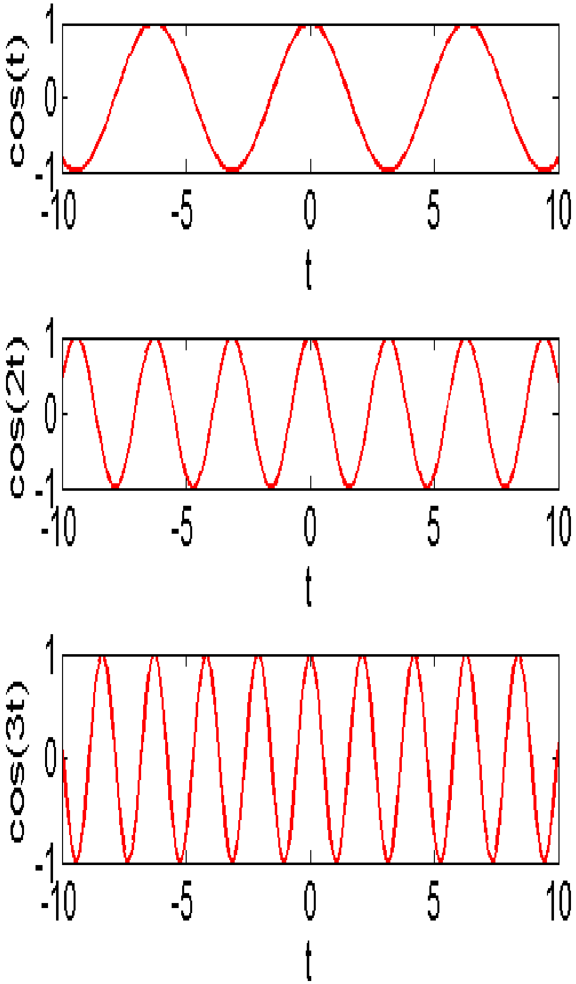
$$E_\infty = \infty$$

Average power:

$$P_{period} = \frac{1}{T_0} E_{period} = 1$$

Useful to consider **harmonic signals**

Terminology is consistent with its use in music, where each frequency is an integer multiple of a fundamental frequency



General Complex Exponential Signals

So far, considered the real and periodic complex exponential

Now consider when *C* can be complex. Let us express *C* is polar form and *a* in rectangular form:

$$C = |C| e^{j\phi}$$

$$a = r + j\omega_0$$

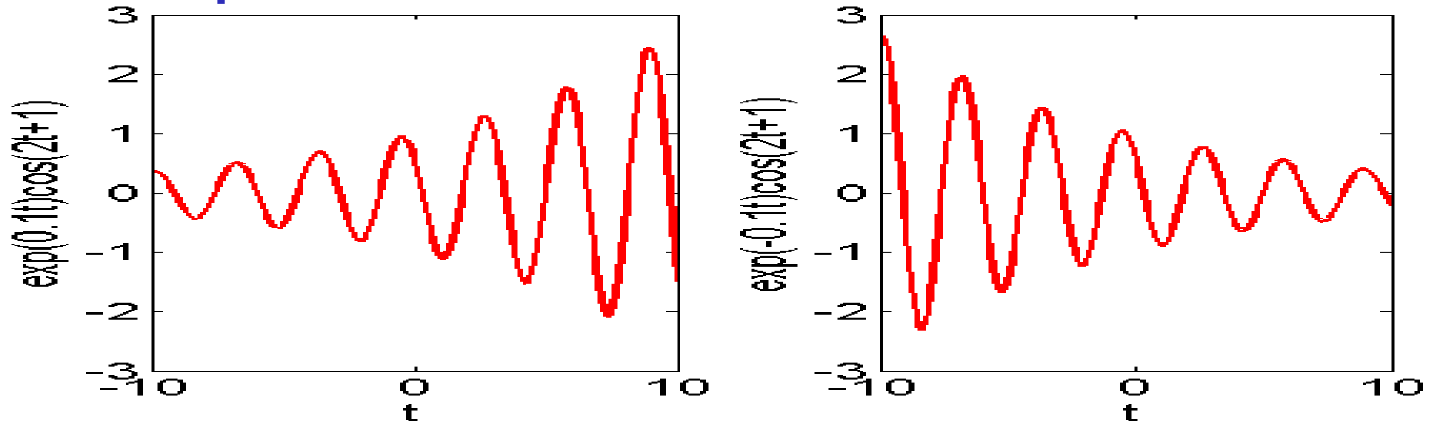
So

$$Ce^{at} = |C| e^{j\phi} e^{(r + j\omega_0)t} = |C| e^{rt} e^{j(\omega_0 + \phi)t}$$

Using Euler's relation

$$Ce^{at} = |C| e^{j\phi} e^{(r + j\omega_0)t} = |C| e^{rt} \cos((\omega_0 + \phi)t) + j|C| e^{rt} \sin((\omega_0 + \phi)t)$$

These are **damped sinusoids**

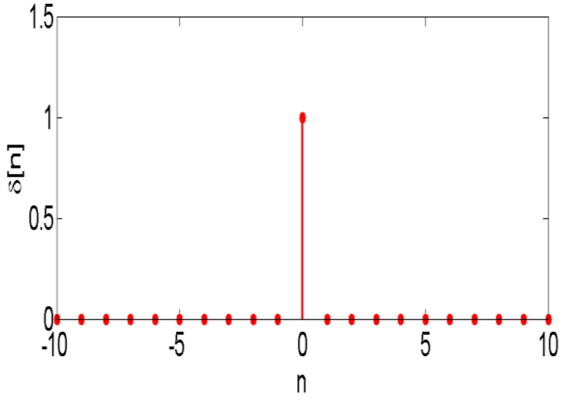


Discrete Unit Impulse and Step Signals

The discrete **unit impulse signal** is defined:

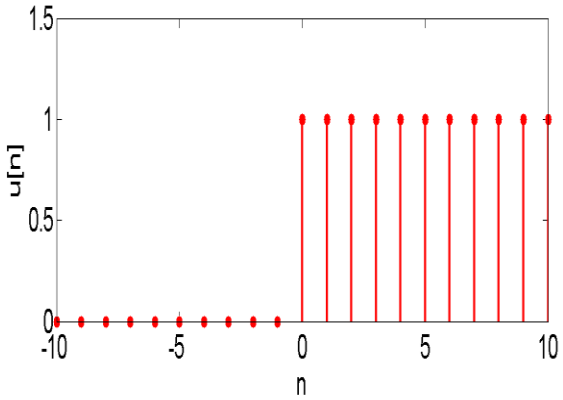
$$x[n] = \delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

Useful as a **basis** for analyzing other signals



The discrete **unit step signal** is defined:

$$x[n] = u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$



Note that the unit impulse is the first difference (derivative) of the step signal

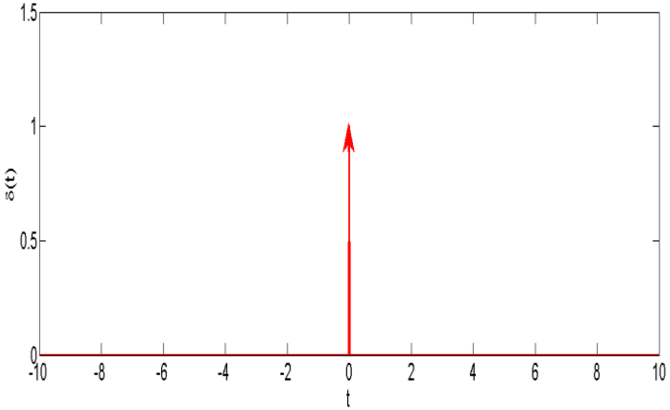
$$\delta[n] = u[n] - u[n-1]$$

Similarly, the unit step is the running sum (integral) of the unit impulse.

Continuous Unit Impulse and Step Signals

The continuous **unit impulse signal** is defined:

$$x(t) = \delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

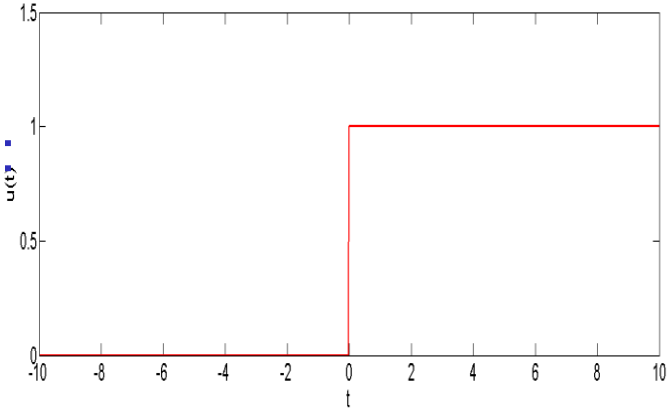


Note that it is discontinuous at *t*=0

The arrow is used to denote area, rather than actual value

Again, useful for an infinite basis

The continuous **unit step signal** is defined:



$$x(t) = u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$

$$x(t) = u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

Lecture 2: Summary

This lecture has looked at signals:

- Power and energy
- Signal transformations
  - Time shift
  - Periodic
  - Even and odd signals
- Exponential and sinusoidal signals
- Unit impulse and step functions

Lecture 2: Exercises

Text:

Q1.3

Q1.7-1.14