EE-232 Signals & Systems

Lecture 3
Signals Concepts & Properties

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Lecture 3: Signals & Systems Concepts

Systems, signals, mathematical models. Continuous-time and discrete-time signals. Energy and power signals. **Linear systems**. Examples for use throughout the course.

Specific objectives:

- Introduction to systems
- Continuous and discrete time systems
- Properties of a system
- Linear (time invariant) LTI systems

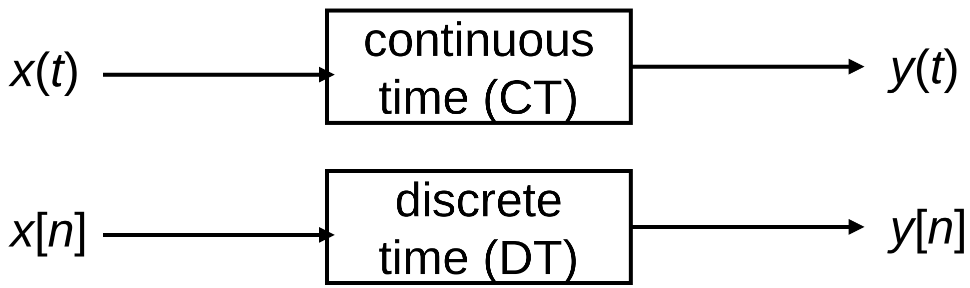
Linear Systems

A system takes a signal as an input and transforms it into another signal

Linear systems play a crucial role in most areas of science

- Closed form solutions often exist
- Theoretical analysis is considerably simplified
- Non-linear systems can often be regarded as linear, for small perturbations, so-called linearization

For the remainder of the lecture/course we're primarily going to be considering Linear, Time Invariant systems (LTI) and consider their properties



Examples of Simple Systems

To get some idea of typical systems (and their properties), consider the electrical circuit example:

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} v_s(t)$$

which is a **first order**, **CT differential** equation.

Examples of **first order**, **DT difference** equations:

$$y[n] = x[n] + 1.01 y[n-1]$$

where *y* is the monthly bank balance, and *x* is monthly net deposit

$$v[n] - \frac{RC}{RC+k} v[n-1] = \frac{k}{RC+k} f[n]$$

which represents a discretised version of the electrical circuit

Example of second order system includes:

$$a \frac{d^2 y(t)}{dt^2} + b \frac{dy(t)}{dt} + cy(t) = x(t)$$

System described by **order** and **parameters** (*a*, *b*, *c*)

First Order Step Responses

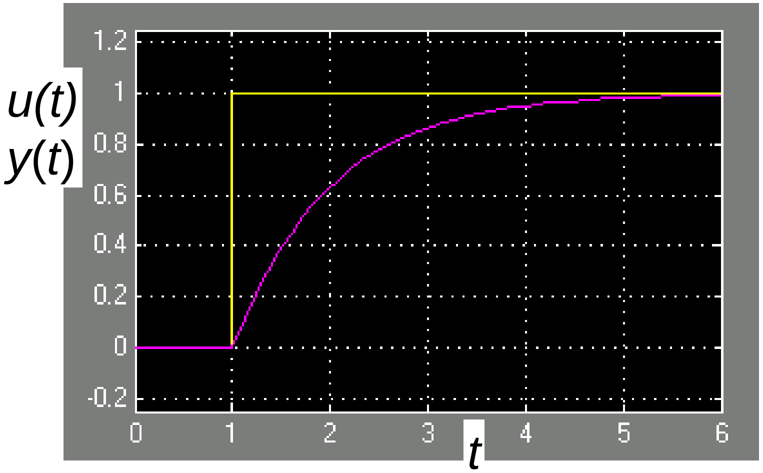
People tend to visualise systems in terms of their responses to simple input signals (see Lecture 4…)

The dynamics of the output signal are determined by the dynamics of the system, if the input signal has no dynamics

Consider when the input signal is a step at *t*, *n* = 1, *y*(0) = 0

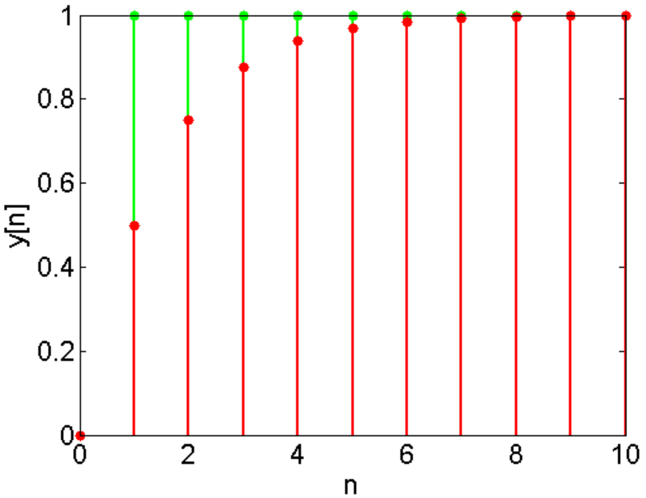
First order CT differential system

$$\frac{dy(t)}{dt} + ay(t) = u(t-1)$$



First order DT difference system

$$y[n](1+ak) - y[n-1] = ku[n-1]$$

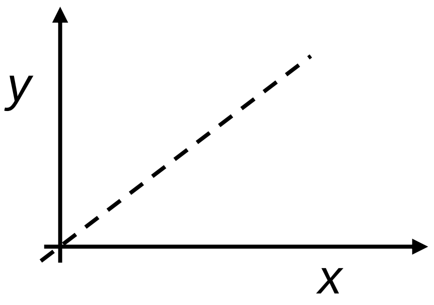


System Linearity

The most important property that a system possesses is **linearity**

It means allows any system response to be analysed as the sum of simpler responses (convolution)

Simplistically, it can be imagined as a line



Specifically, a linear system must satisfy the two properties:

1 Additive: the response to $x_1(t)+x_2(t)$ is $y_1(t) + y_2(t)$

2 Scaling: the response to $ax_1(t)$ is $ay_1(t)$ where $a \in C$

Combined: $ax_1(t)+bx_2(t) \rightarrow ay_1(t) + by_2(t)$

E.g. Linear $y(t) = 3*x(t)$ why?

Non-linear $y(t) = 3*x(t)+2$, $y(t) = 3*x^2(t)$ why?

(equivalent definition for DT systems)

Definition of Time Invariance

A system is time invariant if its behaviour and characteristics are fixed over time

We would expect to get the same results from an input-output experiment, if the same input signal was fed in at a different time

E.g. The following CT system is **time-invariant**

$$y(t) = \sin(x(t))$$

because it is invariant to a time shift, i.e. $x_2(t) = x_1(t\text{-}t_0)$

$$y_2(t) = \sin(x_2(t)) = \sin(x_1(t - t_0)) = y_1(x_1(t - t_0))$$

E.g. The following DT system is **time-varying**

$$y[n] = nx[n]$$

Because the **system parameter** that multiplies the input signal is time varying, this can be verified by substitution

$$x_1[n] = \delta[n] \Rightarrow y_1[n] = 0$$

$$x_2[n] = \delta[n-1] \Rightarrow y_2[n] = \delta[n-1]$$

System with and without Memory

A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent on the output at only that same time (no system dynamics)

$$y[n] = (2x[n] - x^2[n])^2$$

e.g. a resistor is a memoryless CT system where *x*(*t*) is current and *y*(*t*) is the voltage

A DT system with memory is an accumulator (integrator)

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

and a delay

$$y[n] = x[n-1]$$

Roughly speaking, a memory corresponds to a mechanism in the system that retains information about input values other than the current time.

$$y[n] = \sum_{k=-\infty}^{n-1} x[k] + x[n]$$

$$= y[n-1] + x[n]$$

System Causality

A system is causal if the output at any time depends on values of the output at only the present and past times. Referred to as non-anticipative, as the system output does not anticipate future values of the input

If two input signals are the same up to some point t_0/n_0, then the outputs from a causal system must be the same up to then.

E.g. The accumulator system is causal:

$$y[n]=\sum_{k=-\infty}^{n} x[k]$$

because $y[n]$ only depends on $x[n]$, $x[n-1]$, …

E.g. The averaging/filtering system is non-causal

$$y[n]=\frac{1}{2M+1}\sum_{k=-M}^{M} x[n-k]$$

because $y[n]$ depends on $x[n+1]$, $x[n+2]$, …

Most physical systems are causal

System Stability

Informally, a stable system is one in which small input signals lead to responses that do not diverge

If an input signal is bounded, then the output signal must also be bounded, if the system is stable

$$\forall x : |x| < U \rightarrow |y| < V$$

To show a system is stable we have to do it for **all** input signals. To show instability, we just have to find one counterexample

E.g. Consider the DT system of the bank account

$$y[n] = x[n] + 1.01 y[n-1]$$

when $x[n] = \delta[n]$, $y[0] = 0$

This grows without bound, due to 1.01 multiplier. This system is unstable.

E.g. Consider the CT electrical circuit, is stable if $RC>0$, because it dissipates energy

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} v_s(t)$$

Invertible and Inverse Systems

A system is said to be **invertible** if distinct inputs lead to distinct outputs (similar to matrix invertibility)

If a system is invertible, an inverse system exists which, when **cascaded** with the original system, yields an output equal to the input of the first signal

E.g. the CT system is invertible:

$$y(t) = 2x(t)$$

because $w(t) = 0.5*y(t)$ recovers the original signal $x(t)$

E.g. the CT system is not-invertible

$$y(t) = x^2(t)$$

because distinct input signals lead to the same output signal

Widely used as a design principle:

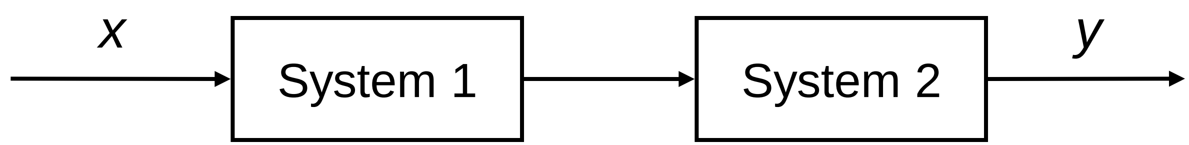
- Encryption, decryption
- System control, where the reference signal is input

System Structures

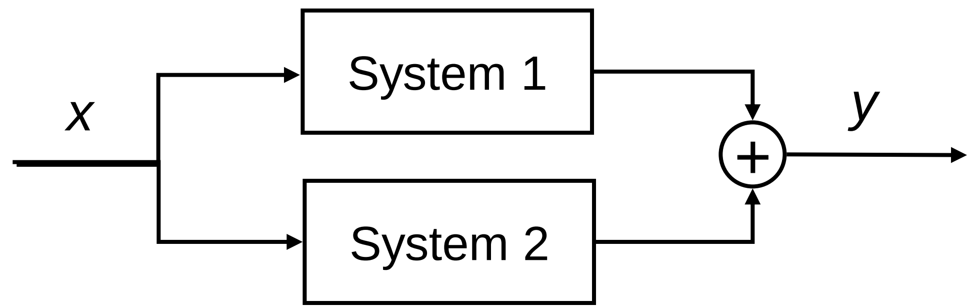
Systems are generally composed of components (sub-systems).

We can use our understanding of the components and their interconnection to understand the operation and behaviour of the overall system

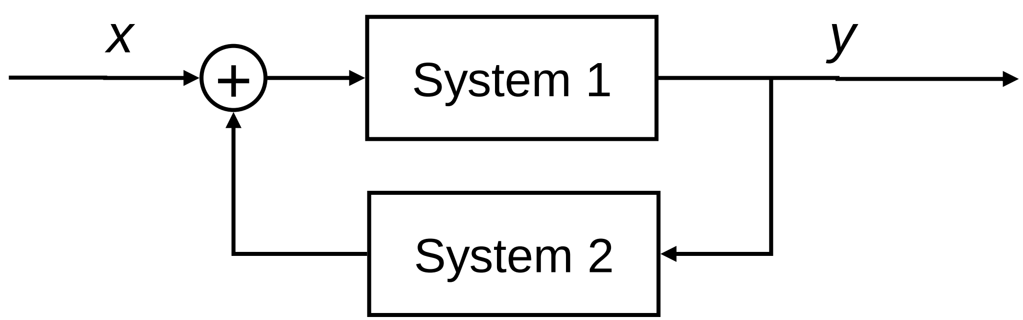
Series/cascade



Parallel



Feedback



Lecture 3: Summary

Whenever we use an equation for a system:

- CT – differential
- DT – difference

The parameters, order and structure represent the system

There are a large class of systems that are linear, time invariant (LTI), these will primarily be studied on this course.

Other system properties such as causality, stability, memory and invertibility will be dealt with on a case by case basis

Matlab and Simulink are standard tools for analysing, designing, simulating complex systems.

Used for system modelling and control design

Lecture 3: Exercises

Sas Q1-27 to Q1-31