EC-310 Microprocessor and Microcontroller Based Design

Number Systems

Nazar Abbas Saqib

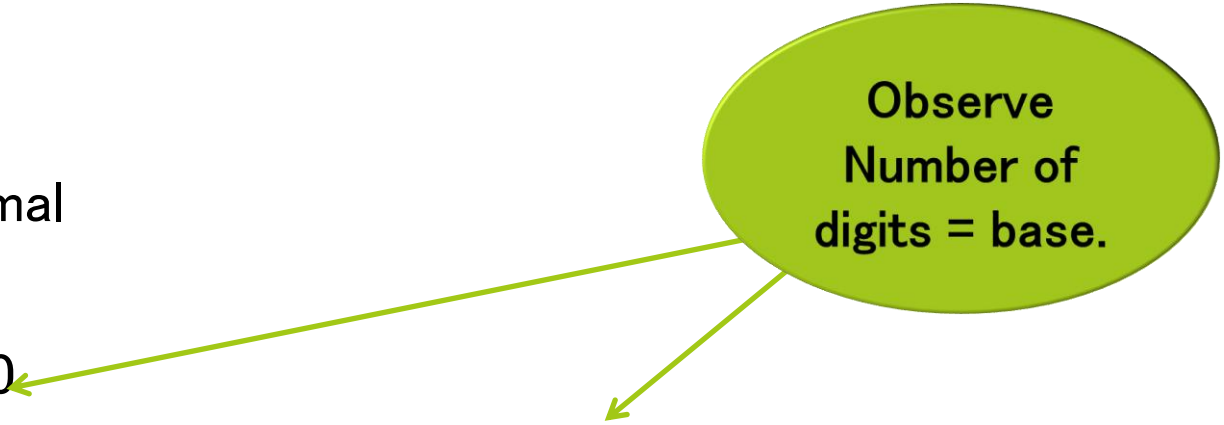
nazar.abbas@ceme.nust.edu.pk

Outline

- ✓ Number System
  - Decimal
  - Binary
  - Octal
  - Hexadecimal
- ✓ Addition , subtraction

Introduction to Number Systems

- Number system is a writing system for expressing numbers
- Human use decimal number system, computers use binary
- Mostly used number systems are:
  - Decimal
  - Binary
  - Octal
  - Hexadecimal
- Decimal
  - Base is 10
  - Consists of digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
  - Minimum = 0 and maximum = base − 1
  - Used by humans in day to day life.



Introduction to Number Systems

- Decimal Number Construction

.....	10^5	10^4	10^3	10^2	10^1	10^0
	100000	10000	1000	100	10	1

- Writing 563 in decimal representation
  - $5x10^2 + 6x10^1 + 3x10^0 = 500 + 60 + 3 = 563$

Introduction to Number Systems

- Binary Number System
  - Base is 2
  - Consists of digits 0, 1
  - Min = 0 and max = base − 1
  - Machine Language

- Conversion from decimal to binary.
  - Divide the decimal number by 2 and write the remainder in right column.
  - Continue this process till quotient becomes zero.
  - Rewrite the remainders in reverse order.

Introduction to Number Systems

- Convert Decimal 25 to binary.

Ex. Convert 25_{10} to binary

		Quotient	Remainder	
25/2	=	12	1	LSB (least significant bit)
12/2	=	6	0	
6/2	=	3	0	
3/2	=	1	1	
1/2	=	0	1	MSB (most significant bit)

Therefore $25_{10} = 11001_2$

Introduction to Number Systems

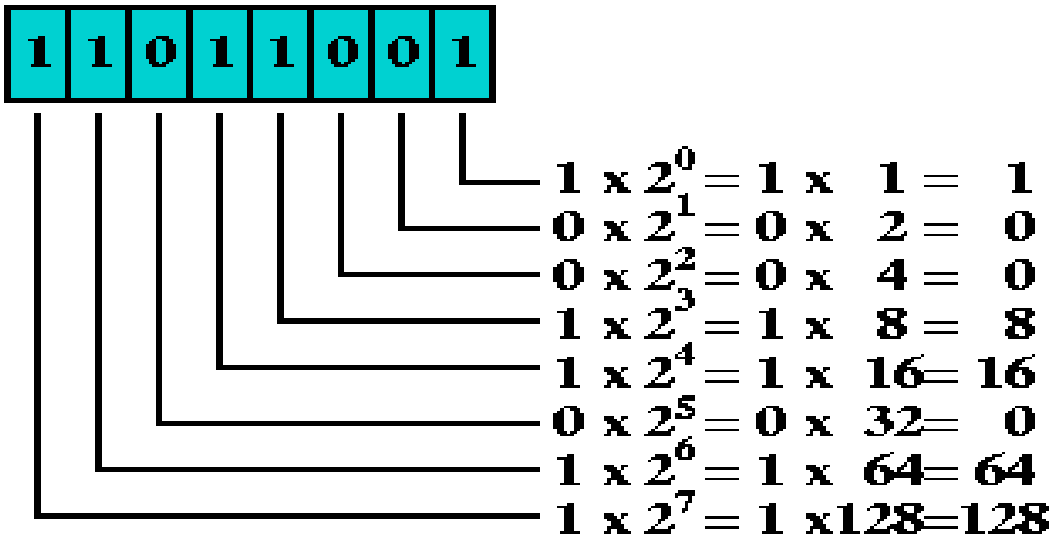
- Writing 11001 in decimal

$1x2^4 + 1x2^3 + 0x2^2 + 0x2^1 + 1x2^0$

$= 16 + 8 + 0 + 0 + 1$

$= 25$

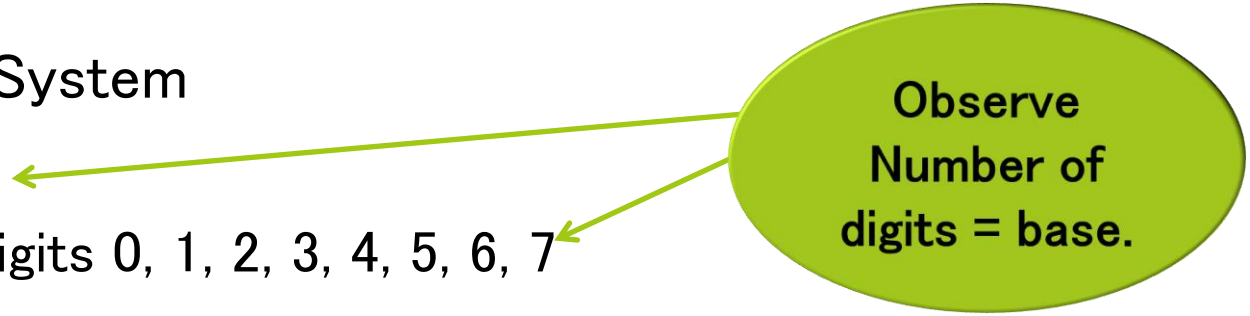
- Writing 11011001 in decimal



$1 + 8 + 16 + 64 + 128 = 217$

Introduction to Number Systems

- Octal Number System
  - Base is 8
  - Consists of digits 0, 1, 2, 3, 4, 5, 6, 7
  - Min = 0 and max = base − 1



- Conversion from decimal to octal.
  - Divide the decimal number by 8 and write the remainder in right column.
  - Continue this process till quotient becomes zero.
  - Rewrite the remainders in reverse order.

Introduction to Number Systems

- Converting 175 to octal

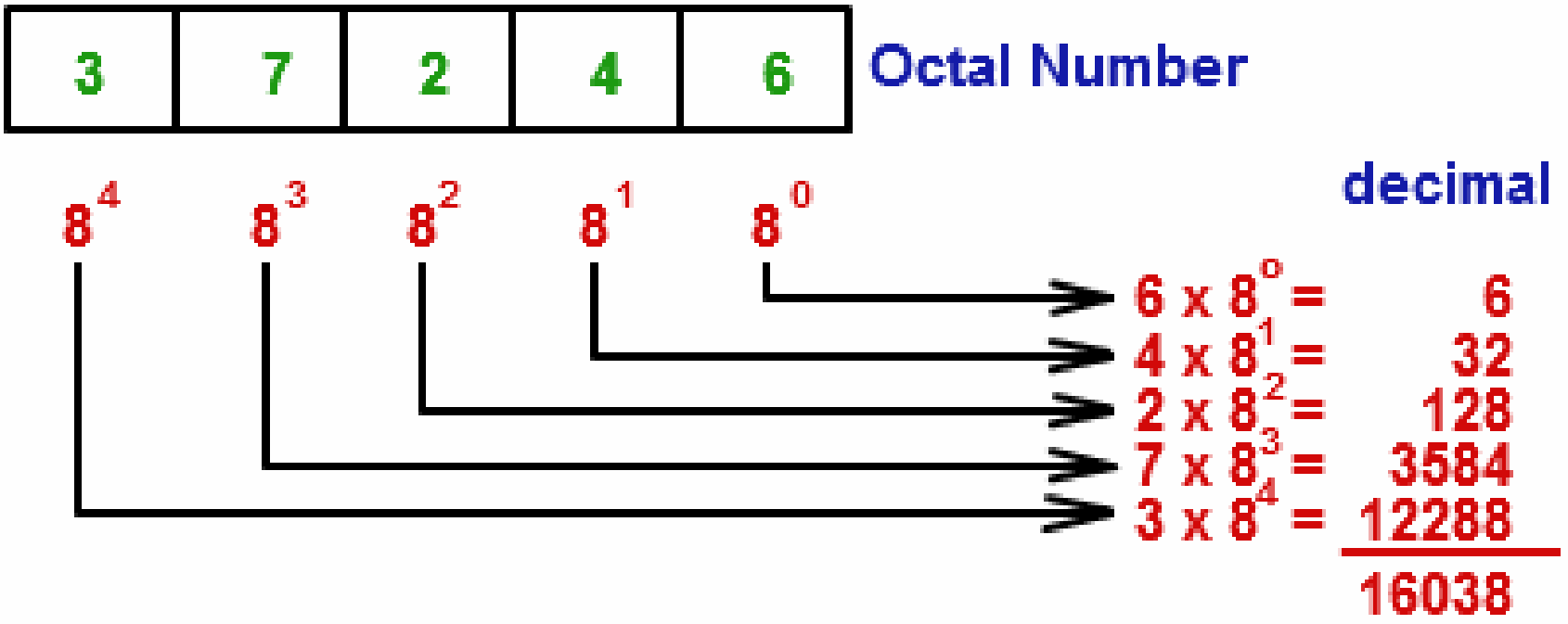
Example: $(175)_{10}$

	Quotient	Remainder	Coefficient
175 / 8 =	21	7	$a_0 = 7$
21 / 8 =	2	5	$a_1 = 5$
2 / 8 =	0	2	$a_2 = 2$

Answer: $(175)_{10} = (a_2\, a_1\, a_0)_8 = (257)_8$

Introduction to Number Systems

- Conversion from octal to decimal



Introduction to Number Systems

- Conversion from octal to decimal

$$122_8$$

$$= (1 \times 8^2) + (2 \times 8^1) + (2 \times 8^0)$$

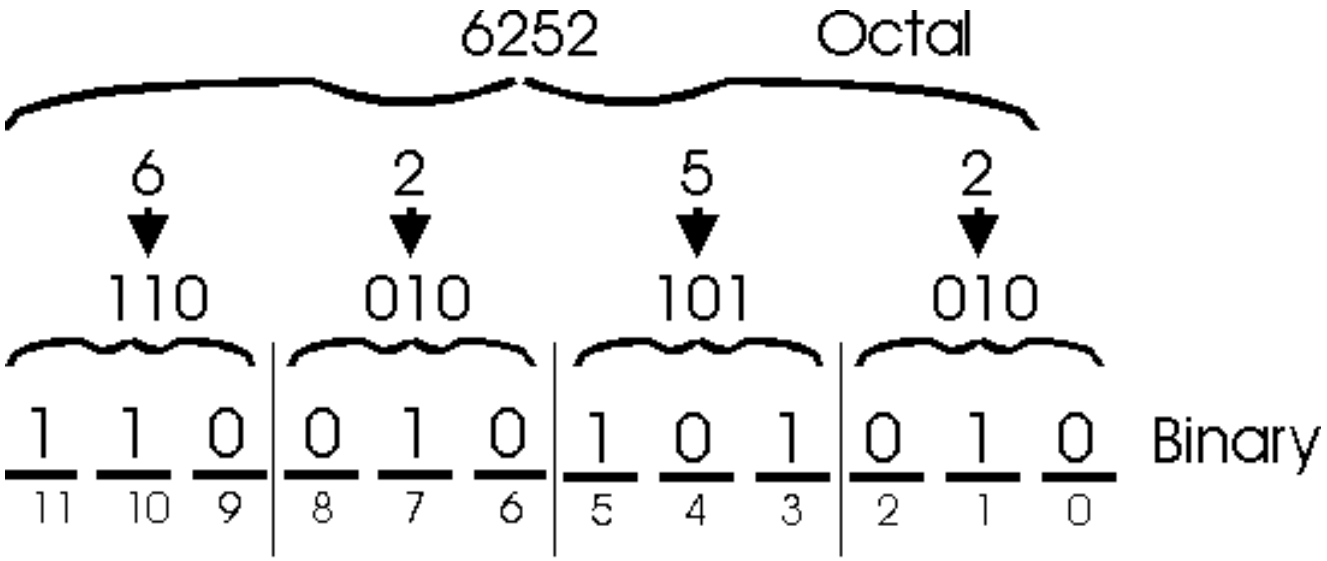
$$= (1 \times 64) + (2 \times 8) + (2 \times 1)$$

$$= 64 + 16 + 2$$

$$= 82_{10}$$

Introduction to Number Systems

- Conversion from octal to binary
  - **Write each octal number in 3-bit binary representation.**



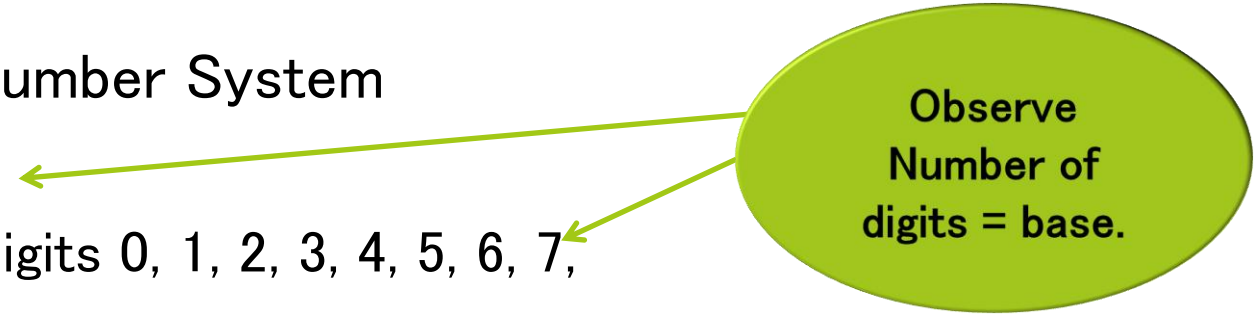
Introduction to Number Systems

- Conversion from binary to octal
  - **Group three bits into single octal number starting from LSB**

Binary	011	100	110	100	010					
Octal	3	4	6	4	2					
Binary	010	101	110	000	001	.	100	110		
Octal	2	5	6	0	1	.	3	6		

Introduction to Number Systems

- Hexadecimal Number System
  - Base is 16
  - Consists of digits 0, 1, 2, 3, 4, 5, 6, 7,
    8, 9, A, B, C, D, E, F
    where A = 10, B = 11, C = 12, D = 13, E = 14, F = 15.
  - Min = 0 and max = base − 1



- Conversion from decimal to hexadecimal.
  - Divide the decimal number by 16 and write the remainder in right column.
  - Continue this process till quotient becomes zero.
  - Rewrite the remainders in reverse order.

Introduction to Number Systems

- Conversion from decimal to hexadecimal.

Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Decimal number 957

Division	Quotient	Remainder
957 / 16	59	0xD
59 / 16	3	0xB
3 / 16	0	0x3

Hexadecimal number 0x03BD

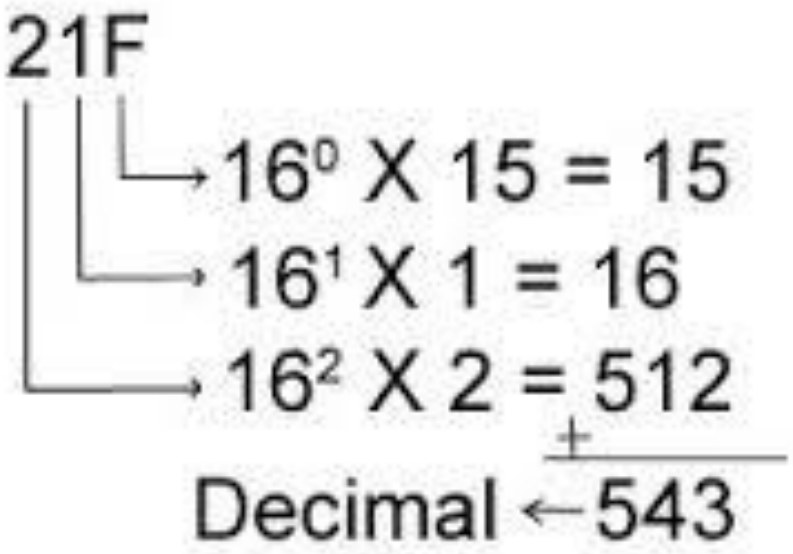
Decimal number 1024

Division	Quotient	Remainder
1024 / 16	64	0x0
64 / 16	4	0x0
4 / 16	0	0x4

Hexadecimal number 0x0400

Introduction to Number Systems

- Conversion from hexadecimal to decimal.



Introduction to Number Systems

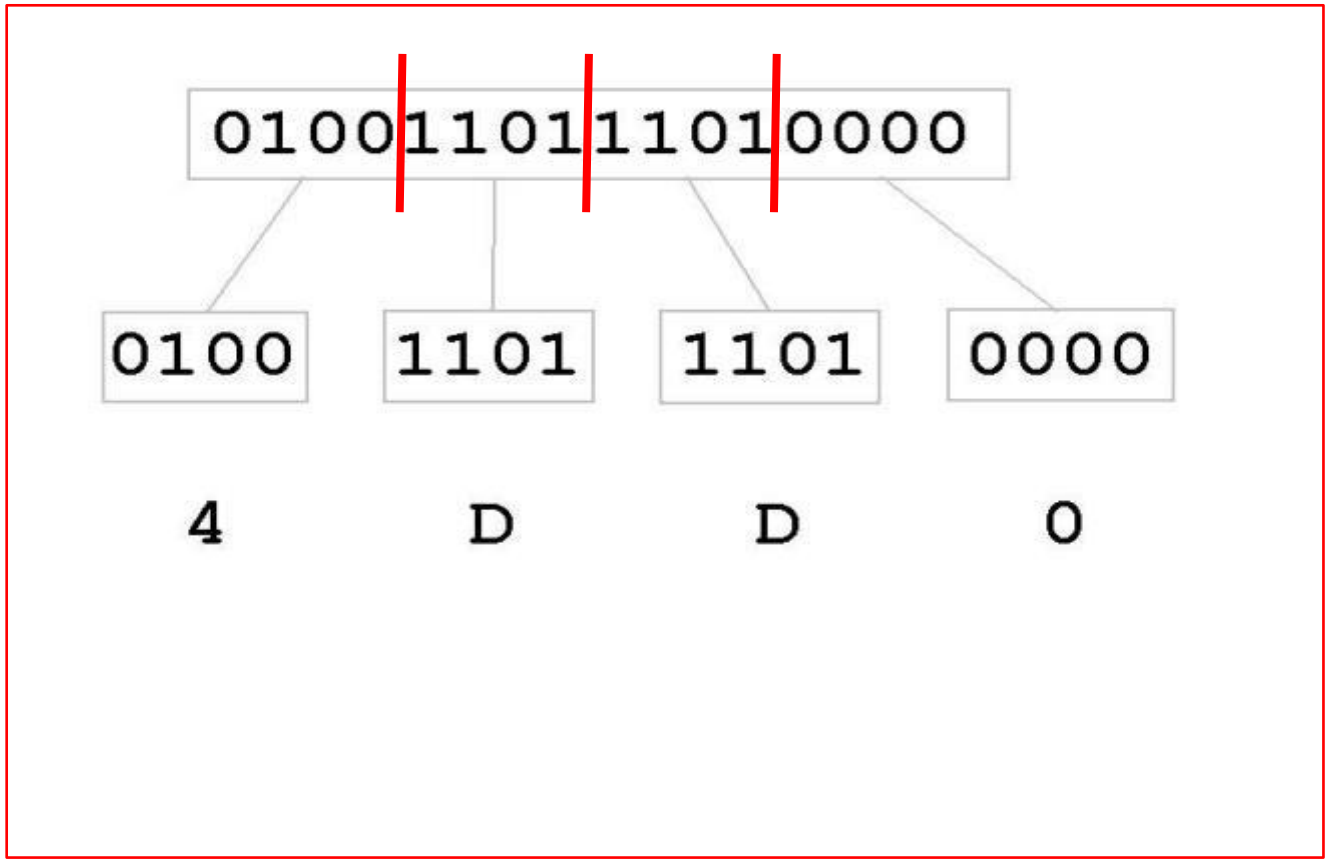
- Conversion from hexadecimal to binary.
  - Write each hexadecimal digit in 4-bit binary representation

Hexadecimal	3	9	A	2
Binary	0011	1001	1010	0010

Hexadecimal	2	D	8	1	.	9	8
Binary	0010	1011	1000	0001	.	1001	1000

Introduction to Number Systems

- Conversion from binary to hexadecimal.
  - Group 4 binary digits into a single hexadecimal digit.



2's Compliment

- For representing negative numbers in binary, 2's compliment representation of a number is used.
- Only Used to represent signed number. MSB is the sign bit.
- Finding 2's compliment involves two steps:
  - Finding 1's compliment of given number.
  - Adding 1 into the LSB of 1's complement.
- 1's compliment is obtained by inverting all the bits.

2's Compliment

Example #1

```
 5 = 00000101
     ↓↓↓↓↓↓↓↓   Complement Digits
     11111010
           +1   Add 1
    ---------
-5 = 11111011
```

Example #2

```
-13 = 11110011
      ↓↓↓↓↓↓↓↓   Complement Digits
      00001100
            +1   Add 1
     ---------
 13 = 00001101
```

Addition of HEX Numbers

- Steps involved in addition of hex numbers:
  - Start from the least significant digit.
  - If sum is less than 16, write down the sum as it is under the numbers.
  - If sum is greater than 16, subtract 16 from sum and write the answer under the two digits being added with a 1 carry to the next significant place.

Addition of HEX Numbers

Ex. Perform hex addition: 23D9 + 94BE

23D9	LSD:	9 + 14 = 23	23 – 16 = 7 w/ carry
+ 94BE		1 + 13 + 11 = 25	25 – 16 = 9 w/ carry
B897		1 + 3 + 4 = 8	
	MSD:	2 + 9 = B	

$4A6_{16} + 1B3_{16} = 659_{16}$

1	carry
4 A 6	$= 1190_{10}$
+ 1 B 3	$= 435_{10}$
6 5 9	$= 1625_{10}$

Subtraction of HEX Numbers

- In subtracting two hex digits, if second digit is greater than first digit, than borrow 16 from the preceding digit.

Ex. Perform hex subtraction: 59F – 2B8

```
  59F     LSD:  15 – 8 = 7
– 2B8           9 + 16 – 11 = 14 = E16
  2E7           5 – 1 – 2 = 2
```

ASCII Numbers

- American Standard Code of Information Interchange.
- Following are assigned ASCII codes
  - Numbers from 0 to 9
  - All alphabets both upper and lower case
  - All punctuation marks
  - Control Codes
- ASCII uses 7 bits to represent code. A 0 is added at the MSB to make it an 8-bit number.
- The sole purpose of this system is to have a common coding scheme for information sharing.